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# Factors Influencing the Forward Acceleration of a Gravity Powered Soapbox Race Vehicle

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***Author's contribution***

*Author designed the study and wrote the first draft of the manuscript.*

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## ABSTRACT

This paper presents the factors influencing and the corresponding derivation of a mathematical model for the forward acceleration of a gravity propelled race vehicle. It can be applied to scale model designs or full size vehicles as typically used in "soapbox Derby" racing events. Study of the model should clarify the main factors and design parameters that influence the acceleration downhill, and deceleration on the level. Maximising the forward acceleration, particularly initially, is a key performance characteristic in reducing the elapsed time of running. The model was used successfully in the design and construction of a full size competition winning vehicle and offers a potentially useful design tool and teaching aid for studies in vehicle dynamics.

*Keywords: Acceleration; soapbox; vehicle.*

## NOTATION

| | |
|---|---|
| $a_X$ | *Forward acceleration in x direction* |
| $A_F$ | *Frontal cross sectional area* |
| $b$ | *Longitudinal distance from rear axle to centre of mass* |

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| | |
|---|---|
| $B_{FX}$, $B_{FY}$ | *Bearing force components, front axle to body* |
| $B_{RX}$, $B_{RY}$ | *Bearing force components, rear axle to body* |
| $C_D$ | *Aerodynamic drag coefficient* |
| $F_D$ | *Aerodynamic drag force* |
| $F_F$, $F_R$ | *Friction resistance force at front wheels, at rear wheels* |
| $g$ | *Acceleration due to gravity* |
| $h$ | *Height of centre of mass from road surface* |
| $I_F$ | *Second moment of mass, front wheels assembly* |
| $I_R$ | *Second moment of mass, rear wheels assembly* |
| $l$ | *Length of wheelbase* |
| $m$, $m_F$, $m_R$ | *Mass of whole vehicle, front wheels, rear wheels* |
| $N_F$, $N_R$ | *Normal reaction from road at front wheels, at rear wheels* |
| $Q_F$, $Q_R$ | *Bearing resistance torque at front axle, at rear axle* |
| $r_F$, $r_R$ | *Rolling radius of front wheels, of rear wheels* |
| $R_F$, $R_R$ | *Rolling resistance force of front wheels, of rear wheels* |
| $v_X$ | *Velocity of vehicle in x direction* |
| $W_B$, $W_F$, $W_R$ | *Weight of body, front wheels, rear wheels* |
| $x$ , $y$ | *Co-ordinate directions, x downhill* |
| $\theta$ | *Angle of road gradient from horizontal* |
| $\mu_R$ | *Coefficient of rolling resistance* |
| $\rho$ | *Density of air* |

## 1. INTRODUCTION

This study concerns the forward acceleration of a gravity powered “soapbox” type vehicle as used in fun and charity contests, and scale model versions for events such as the Pinewood Derby. An expression for acceleration is derived taking account of the various resistances to motion. The analysis is based on a vehicle dynamics teaching model used successfully for many years in Open University (OU) summer schools and reported by Dixon and Martin [1]. In the OU exercise students work in small groups to analyse, design, build and test scale model dragsters each powered by a given spring as the energy source. The tests are timed runs along a 10 m test track and as well as instilling an understanding of vehicle dynamics clearly show the importance of reducing losses due to rolling resistances, friction and aerodynamic drag.

Also taken into account in the summer school exercises are parameters such as wheel diameters, masses and second moments of mass, position of the centre of mass etc. A general observation is that to minimise elapsed time it is more profitable to maximise forward acceleration during the initial stages of the run, even to the extent of coasting unpowered in the latter stages. This situation is often reflected in real drag racing when the winning car is not necessarily travelling faster than its competitor at the finish line. A higher initial acceleration can mean a higher average speed - and hence reduced elapsed time - over the given distance. Acceleration is the key performance factor, as in most forms of motorsport including gravity powered “soapbox” type vehicles.

Coletta and Evans [2] used an energy approach to derive mathematical expressions for elapsed time and instantaneous speed for model vehicles travelling down a track such as is used for the Pinewood Derby event in the USA, making the point that Derby races are often won by no more than one or two hundredths of a second. Mann et al. [3] extended this approach to include a sensitivity analysis of the various parameters identified and made the point that race time and not displacement and velocity is the typical quantity of interest. They also make the point that increasing the vehicle mass results in reduced race times but at a certain limit the increased friction negates this and also state “the most efficient parameter to manipulate is the friction between the wheel and axle.”

The Scottish Cartie Association [4] has tabled a simple model for the acceleration, extended to include an expression for terminal velocity and makes “the slightly surprising observation that the mass and diameter of your wheels has no effect on your terminal velocity” based on an interpretation that the extra inertia of a larger wheel is cancelled out by the larger torque applied to rotate it. This observation is clarified in follow up discussion [5] in observing about wheels that “Moment of inertia does NOT affect your top speed. It only affects the time it take you to get there.”

The Sheffield Hallam University Centre for Sports Engineering Research [6] lodged some general discussion and basic theory in connection with World Record Speed attempts for a gravity powered vehicle. It makes a general point that it is "helpful if we could increase the weight and to try and reduce the rolling resistance in other ways." This was in concluding that the propulsion force due to gravity (i.e. by increasing vehicle mass) has a much greater effect that the loss due to the consequent increase in rolling resistance.

This team and other references generally agree that higher tyre pressures lead to less rolling resistance due to less energy being lost in deformation. The team makes the point that this applies on good surfaces but in the event of disturbances such as potholes some allowable tyre deformation at that instance will save some vehicle energy. Thus, for every given wheel and road surface combination, "there is a sweet spot of optimum pressure."

The same team [7] also lodged an introduction and commentary on the design constraints they worked to and the decision to build the record vehicle to "a maximum weight (including the driver) of 200 kg".

In terms of the practical physics involved, Gale [8] has published a 52 page document containing many recommendations and tips on design, construction and indeed driving of gravity powered vehicles. The approach reported herein identifies the parameters which affect the acceleration and it is hoped can provide an understanding of basic vehicle dynamics which along with the aforementioned references may provide a useful guide for designing gravity powered vehicles, whether of full or scale model sizes.

## 2. SETTING THE SCENE

The Pinewood Derby event has been run since 1953 and literally millions of pupils and students have benefitted in terms of understanding dynamics and in the practical issues associated with applying theory to real models or products. Events for full size vehicles have become popular too in this period and continue to introduce youngsters to the fun, trials and tribulations of motorsport – including preparation, dealing with entries and officials, passing through tech./scrutineering etc. Fig. 1 shows a typical paddock scene from a recent charity event in the UK, reminiscent of a European Formula 2 race meeting in the 1960's!


**Fig. 1. A typical paddock scene, getting ready for the event**


**Fig. 2. A close finish? Typical "soap box" carts in action (Courtesy Wikimedia Foundation [9])**

Generally speaking events are run fairly informally, but having issues associated with or which parallel "real" motorsport events. Vehicles can be run in pairs (as with drag racing), in groups, singly, or at staged intervals, depending on the track, hill or starting ramp. Usually there are several elimination heats culminating in a final. Vehicles are released at the start at the top of the suitable hill or special starting ramp, and to ensure consistency and fairness push or running starts are often forbidden. Thus gravity is the only power source. As mentioned above, performances can be remarkably close at the finish line, as indicated in Fig. 2 above.

Such closeness implies that it is important to pay attention to the details that may affect the performance, however small. Key to this is the initial acceleration and any final deceleration if the track levels out, thus it is important to consider all the factors of influence.

## 3. FREE BODY DIAGRAMS AND ANALYSIS MODEL

The OU has long adopted a procedural approach to solving dynamics problems as reported by Dixon and Martin [1] and a key step is formulating the appropriate free body diagrams and a reference axis system. Fig. 3 shows the free body diagrams of the chassis/body (including driver) and the front and rear wheel assemblies. There are normally 4 wheels but for the purposes of analysis the two front wheels, any axles and other rotating parts are considered as one rotating mass, as are the two rear wheel assemblies. The vehicle is rolling down the slope and it is assumed that there is no slip at the wheel to road contact.

The only force providing a positive acceleration in the $x$ direction is the weight component, $mg \sin\theta$, which may be why some teams opt for heavier constructions. Some of the resistance forces and other issues are also directly or indirectly a function of the mass however so it is not as clear cut. For example, $Q_F$ and $Q_R$ are the resistance torques due to the bearings and any other friction in the rotating parts. These torques will almost certainly increase as the mass increases even though often in conventional road vehicle analysis they are neglected. The torques also do negative work sapping energy from the vehicle. The friction forces $F_F$ and $F_R$ at the tyre to road contact patches are generated in response to $Q_F$ and $Q_R$ *and* to provide the rotational acceleration of the wheel assemblies. Thus during the acceleration stages $F_F r_F \neq Q_F$ and $F_R r_R \neq Q_R$ but they do no work on the vehicle unless there is slipping or skidding between the wheels and road surface.

The forces $R_F$ and $R_R$ are separate rolling resistance forces due to the tyre distortion and possible scrub effects at the contact patch. They are difficult to quantify and generally relate to the normal reactions by a rolling resistance coefficient such that $R_F = \mu_R N_F$ and $R_R = \mu_R N_R$ where $\mu_R$ takes values typically in the region 0.01 to 0.02 for pneumatic tyres on asphalt. It should be noted that $N_F$ and $N_R$ are in turn functions of the vehicle mass and that both $R_F$ and $R_R$ also do negative work on the vehicle but have no moment about the wheel axes. In this respect these rolling resistance forces are sometimes modelled as acting at wheel centre height or alternatively so as to bring forward slightly the normal reactions $N_F$ and $N_R$ so that the resultants pass through the wheel centres. It is important

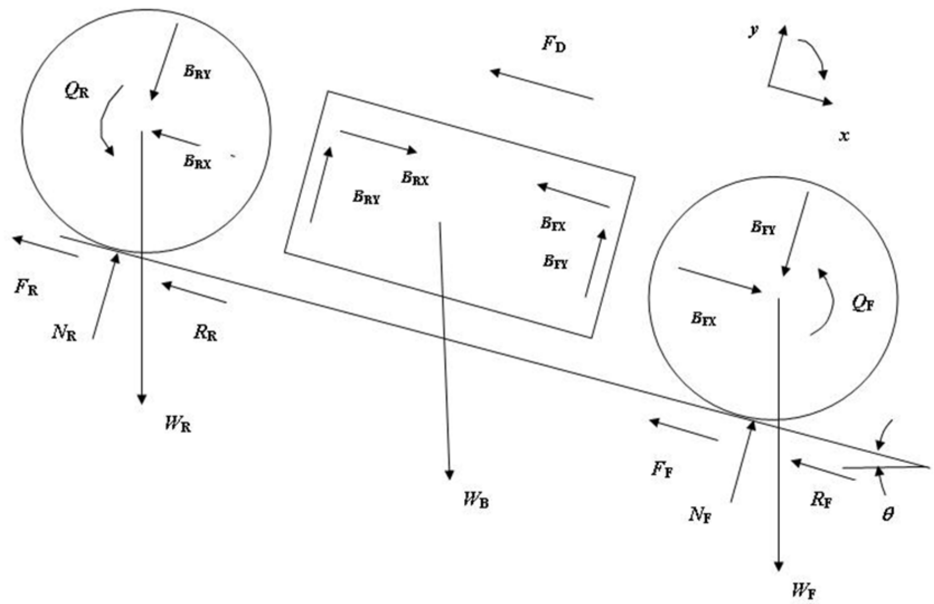


**Fig. 3. Free body diagrams of a gravity propelled vehicle**

to distinguish between the two types of rolling resistance forces therefore, particularly for "soapbox" type vehicles where they may be significant enough to warrant attention.

The aerodynamic drag force will come into play as the vehicle speed increases and is given by the usual format:

$$F_D = \frac{1}{2} C_D \; A_F \; \rho \; v_X^{\,2} \qquad (1)$$

The variables under control are the drag coefficient, $C_D$, and frontal cross sectional area, $A_F$.

The equations of motion are:

$$W \sin\theta - F_R - R_R - F_F - R_F - F_D = m a_x \qquad (2)$$

$$F_F \; r_F - Q_F = I_F \; a_X / r_F \qquad (3)$$

$$F_R r_R - Q_R = I_R \; a_X / r_R \qquad (4)$$

Combining Eq. (2), (3) and (4) the forward acceleration of the vehicle is given by:

$$a_x = \frac{mg\sin\theta - R_F - R_R - F_D - \dfrac{Q_F}{r_F} - \dfrac{Q_R}{r_R}}{m + \dfrac{I_F}{r_F^{\,2}} + \dfrac{I_R}{r_R^{\,2}}} \qquad (5)$$

## 4. COMPUTER MODEL AND STUDY

A computer program in was written in Fortran 95, tested and verified and used to study the effects on predicted forward acceleration values of $a_X$ for ranges of the various design parameters. These were mainly the vehicle total mass, $m$, the masses of the wheels, $m_F$ and $m_R$, the rolling radii of the wheels $r_F$ and $r_R$, the wheelbase, $l$, and the position of the centre of mass as given by $b$ and $h$. The Appendix gives the ranges and particular values chosen for the study. For this type of vehicle it was assumed that the rolling radius of a loaded wheel was negligibly different from the unloaded radius, so the rolling radius was simply half the nominal diameter.

From the initial studies a baseline specification was chosen and the main parameters studied in turn. The base vehicle data were: total mass 60 kg, mass per wheel 0.5 kg, diameter of wheels 1.0 m, height of centre of mass 0.4 m, position of centre of mass ahead of rear axle 0.7 m, wheelbase 2.0 m.

As would be expected, decreasing the bearing resistance torques (including the number of wheels) had a small positive linear effect on the forward acceleration. Initial acceleration was not affected by frontal cross sectional area or drag coefficient as the aerodynamic drag is of course velocity dependent. For the base specification vehicle the aerodynamic drag force at a speed of

22 km $h^{-1}$ was of the order 7.0 N. The position of the centre of mass and wheelbase had very little effect on overall acceleration.

In terms of design the major features that had an effect on acceleration were the total mass, the masses of the wheels and the diameter of the wheels (i.e. the rolling radius). Fig. 4 shows that for a total mass in the range of around 60 to 120 kg, the acceleration increases as mass increases, but in a reducing non-linear relationship as the rolling resistance force will also increase. The bearing resistance force will also increase slightly, thus it is profitable to reduce as far as possible both these forces, for example with small diameter and freely rolling precision wheel bearings and with small section tyres at higher pressures. Note also the beneficial effect of reducing the number of wheels on one axle from 2 to 1, where the rules permit. If push starts are not allowed, as is now often the case, there is little point in reducing total mass to below about 60 kg.

Fig. 5 shows however that there is an incentive for reducing the masses of the individual wheels for a given rolling radius. This quite clearly reflects the need for energy to accelerate the rotating parts about their own axes. It could be argued that the opposite would be the case in a long run with for example a fairly horizontal surface section. The heavier wheel in this case would act as an energy store and reduce the negative acceleration. Generally though, as with the OU dragster project, it is better to harness as much positive acceleration as possible early on to reduce overall average speeds. It depends on the layout and topography of the race track.

Fig. 6 shows that the wheel diameter (i.e. the rolling radius) is a crucial parameter. For a given wheel mass a diameter in the range of about 0.5 to 1.0 metres would seem to be advantageous. This reflects the frictional drag force due to the bearing resistances being reduced as the rolling radius increases, even though the second moment of mass will also increase and negate some of this gain. It would seem therefore that a good solution would be a large diameter but light weight wheel running on a fixed axle or spindle and supported by small freely running bearings.

It is a simple matter to use the mathematical model to predict the straight line acceleration downhill but some general observations can be made:

- The vehicle must have a low aerodynamic drag, i.e. a low drag coefficient and low frontal cross sectional area;
- The wheel bearings must be of a small diameter and as freely rolling as possible to minimise the resistance torques;
- The wheel and tyre must have minimum rolling resistance at the road surface contact patch, e.g. perfectly aligned for minimum scrub and (if appropriate) with highest recommended tyre pressures suitable for the given track surfaces;
- The wheels must be of a large diameter but at the same time be as light as possible;
- The overall vehicle mass is not critical and, within limits, a higher mass is better.

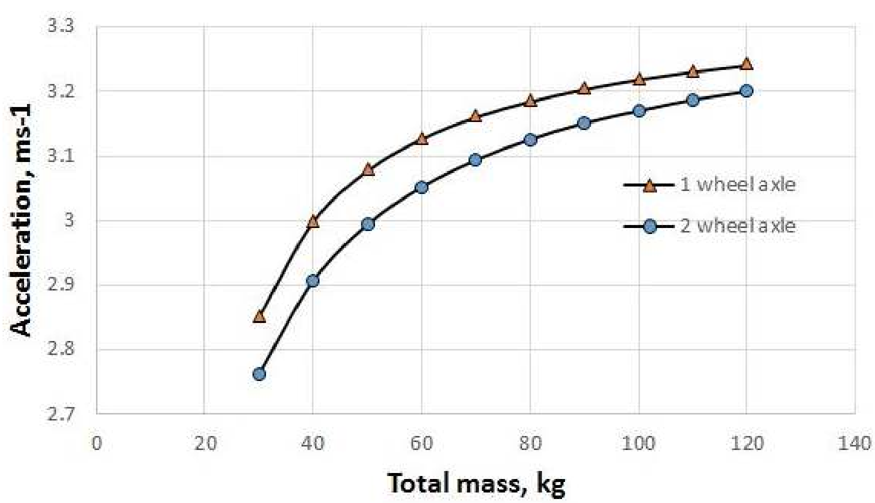


**Fig. 4. Acceleration vs. total vehicle mass**

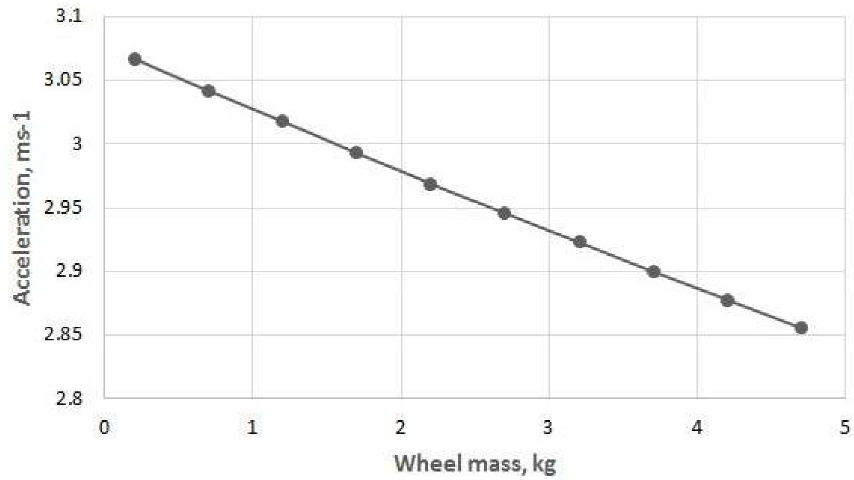


**Fig. 5. Acceleration vs. individual wheel mass**

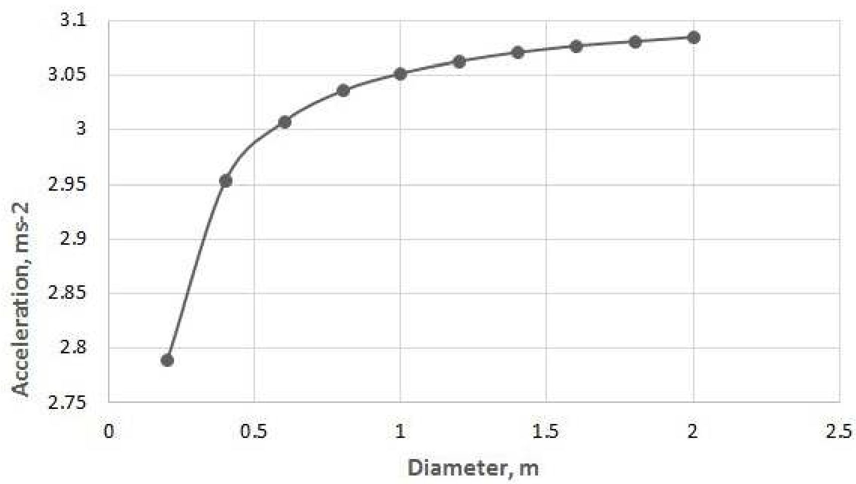


**Fig. 6. Acceleration vs. wheel diameter**

The model was used to guide the design and construction of a real vehicle shown in Fig. 7. This vehicle was driven by the 10 year old grandson of the constructor in the annual Newport Pagnell Carnival soap box Derby in the UK, winning the event outright. The vehicles ran down a hill on one of the town's streets (closed officially to normal traffic!) and competed in pairs in heats and a final and were powered solely by gravity (i.e. no push starts). For the winning team this focussed attention on the initial acceleration requirements. In particular attention was paid to reducing both types of rolling resistance forces by having large diameter but light wheels, reduced to 3 in number, with high pressure tyres; having a low frontal cross section area and an attempt at a low drag shape. Further improvements to these aspects are already being considered for future competitions.

**Fig. 7. An event winning soapbox vehicle**

## 5. CONCLUSION

A mathematical model has been formulated to express the straight line acceleration of a gravity powered vehicle. Observations are given regarding the factors which influence the acceleration. These factors can be evaluated and applied to the model systematically to arrive at an optimum design. The model was considered in the design of a successful event winning vehicle. Observations on the factors which improve acceleration and thereby overall performance generally agree with those of Coletta and Evans [2] and Mann et al [3] using energy based approaches. It is hoped therefore that this paper will add to the understanding of such factors and provide useful guidelines for the design of future gravity propelled vehicles.

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## COMPETING INTERESTS

Author has declared that no competing interests exist.

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# APPENDIX

## GENERAL INPUT DATA:

Acceleration due to gravity, 9.81 $ms^{-1}$
Air density, 1.225 kg $m^{-3}$
Angle of slope, 21 degrees

## CHASSIS INPUT DATA:

Drag coefficient, 0.60
Frontal cross sectional area, 0.50 $m^2$
Mass, total, min. 30 kg, max. 120 kg
Position of centre of mass, height, min. 0.1 m, max. 0.4 m
Position of centre of mass, from rear axle, min. 0.4 m, max. 1.6 m
Wheelbase, min. 1.2 m, max. 2.8 m

## WHEELS INPUT DATA:

Bearing resistance torque per axle, 2.0 Nm
Mass per wheel, min. 0.2 kg, max. 4.7 kg
Number of wheels per axle, min.1, max. 2
Rolling radius, min. 0.1 m, max. 1.1 m
Rolling resistance coefficient 0.018.

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