

Innovative Techniques Unveiled in Advanced Sheet Pile Curtain Design

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Abstract

This thorough review explores the complexities of geotechnical engineering, emphasizing soil-structure interaction (SSI). The investigation centers on sheet pile design, examining two primary methodologies: Limit Equilibrium Methods (LEM) and Soil-Structure Interaction Methods (SSIM). While LEM methods, grounded in classical principles, provide valuable insights for preliminary design considerations, they may encounter limitations in addressing real-world complexities. In contrast, SSIM methods, including the SSI-SR approach, introduce precision and depth to the field. By employing numerical techniques such as Finite Element (FE) and Finite Difference (FD) analyses, these methods enable engineers to navigate the dynamics of soil-structure interaction. The exploration extends to SSI-FE, highlighting its essential role in civil engineering. By integrating Finite Element analysis with considerations for soil-structure interaction, the SSI-FE method offers a holistic understanding of how structures dynamically interact with their geotechnical environment. Throughout this exploration, the study dissects critical components governing SSIM methods, providing engineers with tools to navigate the intricate landscape of geotechnical design. The study acknowledges the significance of the Mohr-Coulomb constitutive model while recognizing its limitations, and guiding practitioners toward informed decision-making in geotechnical analyses. As the article concludes, it underscores the importance of continuous learning and innovation for the future of geotechnical engineering. With advancing technology and an evolving understanding of soil-structure interaction, the study remains committed to ensuring the safety, stability, and efficiency of geotechnical structures through cutting-edge design and analysis techniques.

Keywords

Sheet Pile Curtain Design, Soil-Structure Interaction, Geotechnical Engineering, Advanced Design Techniques, Finite Element Analysis, Innovative Geotechnical Methods

1. Introduction

Embarking on a journey into the intricacies of geotechnical engineering, this article represents both an exploration and a groundbreaking study titled “Innovative Techniques Unveiled in Advanced Sheet Pile Curtain Design.” In the dynamic realm of structural design and simulation, the crucial significance of computational models in accurately replicating the intricate interplay between structures, site conditions, and the surrounding environment cannot be overstated. This complex interrelation, commonly denoted as soil-structure interaction (SSI), stands as the bedrock of modern geotechnical engineering.

Sheet piles, unsung heroes of civil engineering, constitute the sturdy backbone of retaining walls, cofferdams, and various geotechnical structures engineered to withstand formidable earth pressures. The precision and accuracy with which these sheet piles are designed are key to ensuring the safety, stability, and effectiveness of these vital structures. This article embarks on an enlightening journey, demystifying the complex world of sheet pile design while shedding light on the distinct categories of methods used for this purpose, each offering unique insights into the intricate interplay between structural components and the encompassing soil.

Within the realm of sheet pile design, engineers commonly wield two broad approaches: Limit Equilibrium Methods (LEM) and Soil-Structure Interaction Methods (SSIM). The former, known for their analytical simplicity, often find their place in the early stages of design. These classical techniques, steeped in the wisdom of pioneering engineers like Coulomb, Rankine, and Boussinesq, assess sheet pile stability by pinpointing conditions at the verge of failure while assuming plastic soil behavior. Although valuable for initial design considerations, LEM methods may fall short in capturing the nuances of real-world soil-structure interaction.

In stark contrast, SSIM methods herald a paradigm shift in sheet pile design. These numerical techniques, epitomized by the SSI-SR (Soil-Structure Interaction with Springs and Reaction Laws) method, usher in a new era of comprehensive understanding regarding how sheet piles interact with their geotechnical surroundings. Grounded in the principles of continuum mechanics, SSIM methods employ advanced tools like Finite Element (FE) and Finite Difference (FD) analysis to meticulously model the intricate behaviors exhibited by both soils and structures. This article’s spotlight remains firmly fixed on SSIM numerical methods, celebrated for their capacity to account for the intricacies of

soil-structure interaction and deliver unparalleled precision in sheet pile design.

As this study delves deeper into the world of geotechnical engineering, the Finite Element Method (FEM) takes center stage. Within this powerful computational technique lies the Soil-Structure Interaction Finite Element (SSI-FE) method, a revered tool in civil engineering. This method empowers engineers to explore the intricate interactions between structures and the encompassing soil, offering invaluable insights into design, construction, and safety assessments. The present article embarks on a journey to unravel the principles and applications of the SSI-FE method, with a special focus on its role in modeling retaining walls. Our aim is to illuminate how this method elevates engineering precision, particularly in scenarios where the interaction between structures and soil is pivotal.

But the present exploration doesn't stop there. Within the realm of structural analysis, the section titled "Three-dimensional calculation of sheet piles—Eurocode 7 and methods for sheet pile wall analysis" takes us on a captivating journey into the world of sheet pile walls. Despite the leaps in computational technology, the adoption of three-dimensional analyses in practical scenarios, particularly for retaining structures, has remained somewhat restrained. The article confronts this limitation head-on and casts light on scenarios where three-dimensional models, as exemplified by the work of Chehade *et al.* and Popa *et al.*, prove to be invaluable, especially in the intricate urban settings and deep excavations that define modern engineering challenges.

The subsequent sections embark on an expedition through the intricate domain of geotechnical engineering, where the convergence of innovation and precision unfolds to shape the future landscape of sheet pile curtain design.

2. Leading-Edge in Advanced Sheet Pile Curtain Design

The comprehension and simulation of the behavior of geotechnical structures predominantly rely on the capability of the computational model to accurately replicate the physical behavior of the structure, the site conditions (soil characteristics and environmental aggressiveness), and ultimately, the interaction of the structure with its surroundings, known as soil-structure interaction (SSI). In practical terms, the design of geotechnical structures, particularly sheet pile curtains, is rooted in a semi-deterministic approach. This design process is carried out using either analytical methods or precise numerical techniques, which demand meticulous input data and calculation assumptions. Furthermore, various types of uncertainties, whether they are of a random or epistemic nature, are addressed through predefined partial safety factors.

2.1. Sheet Pile Retaining Structure

Among the various available structural support solutions, consideration can be given to employing sheet pile type retaining walls. Initially, in the early part of the 1900s, the preference was for using reinforced concrete to create the initial

sheet pile designs, primarily due to economic factors. However, this approach encountered certain drawbacks associated with the installation process, the adjustment of sheet pile components, and the frequent occurrence of issues like cracks [1] [2]. As time progressed, notably from the 1960s onwards, there was a substantial rise in the adoption of diverse profiles of metallic sheet piles. This shift was driven by both technological advancements, specifically industrialization, and the concurrent increase in global steel production, which in turn led to a reduction in production costs within Western Europe.

In modern engineering practices, steel sheet piles are frequently employed in marine or river environments, such as quaysides or riverbanks, primarily due to their excellent waterproofing capabilities. They are also adaptable to various configurations based on project requirements, including locations near railways or within urban areas. Generally, the utilization of sheet piles offers an attractive balance between cost and quality, largely attributed to their efficient factory fabrication process, which results in swift installation.

Concerning quality control, the manufacturing process ensures continuous and accessible quality checks during fabrication, a contrast to on-site construction where this becomes intricate due to site-specific conditions, geological variations, structural complexities, and technological limitations. However, the adoption of sheet pile retaining systems comes with challenges. The installation procedures, involving methods like driving or sinking, encounter difficulties in rocky terrains or dense sands, resulting in significant noise disturbances in urban zones. Additionally, transportation poses limitations, restricting the maximum structure length to 24 meters, due to road safety regulations and accessibility issues [3].

In addition to the diverse technological approaches employed, which can vary depending on the specific type of construction, the mechanical properties, especially the structural rigidity, play a pivotal role in distinguishing between different categories of retaining structures. In 2003, Dawkins *et al.* introduced a classification that facilitates a comparative assessment of various retaining systems, including sheet pile curtains (comprising interconnected sheet piles forming a continuous retaining barrier), and other solutions such as secant piles and diaphragm walls [4]. This assessment is rooted in the structural stiffness or flexibility, represented by the bending stiffness parameter EI . This classification effectively categorizes these structures into two groups: flexible and rigid systems (**Figure 1** and **Table 1**).

The question that arises is whether a true differentiation exists between these two categories of structures (flexible and rigid), or in other words: does this classification effectively delineate two distinct response behaviors of the structure? The unequivocal answer to this query is in the affirmative. Just like any geotechnical structure, the interplay between soil and structure (SSI) is dictated by the inherent rigidity of both the structure itself and the encompassing soil. Assuming that a structure leans towards being either flexible or rigid results in the

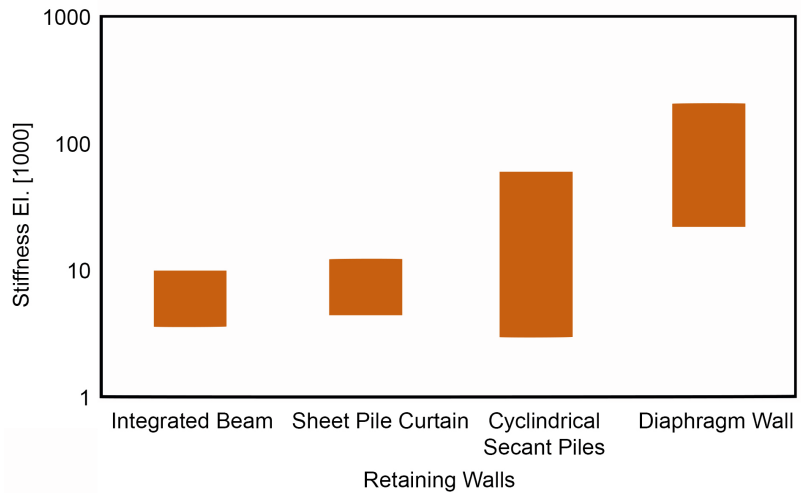


Figure 1. Comparison of flexural stiffness of various types of retaining structures built from [4].

Table 1. Classification of retaining structures based on stiffness [4].

Types	Retaining structure	EI [kN·m ² /m] × 10 ⁴
Flexible	Integrated Beam System	4.16 - 8.22
Flexible	Vertical Sheet Pile Curtain	3.35 - 6.50
Rigid	Cylindrical Secant Piles	2.87 - 52.78
Rigid	Diaphragm Walls	21.90 - 175.00

initiation of distinct mechanisms. This phenomenon is well-established for mat foundations, where it is commonly acknowledged that a higher degree of flexibility in the soil-structure system accentuates the manifestations of SSI effects, and conversely, a greater rigidity dampens the observable impact of SSI.

In 2001, Delattre proposed that this principle extends to retaining structures as well, where the displacement of a firm retaining wall corresponds to a pivotal motion at the base of the structure considering embankment retaining, or at the summit within the framework of excavation support (**Figure 2**) [5]. In **Figure 2** ① represents the motion of the Retaining Wall through Rotation, ② is the Lateral Decompression and Settlement of the Supported Soil, ③ is the Lateral Compression and Uplift of the Soil in Front of the Retaining Wall, ④ represent Slight Lateral Decompression of the Supported Soil Near the Support, ⑤ represent the Significant lateral decompression of the supported soil beneath the support, ⑥ represent the Lateral compression and uplift of the ground in front of the upper part of the sheet pile, and ⑦ is the Counterfort.

2.2. Sheet Pile Curtains and Their Behavior

Moreover, the behavior of flexible structures diverges notably. The inherent flexibility leads to planar deformations that predominantly influence the lateral

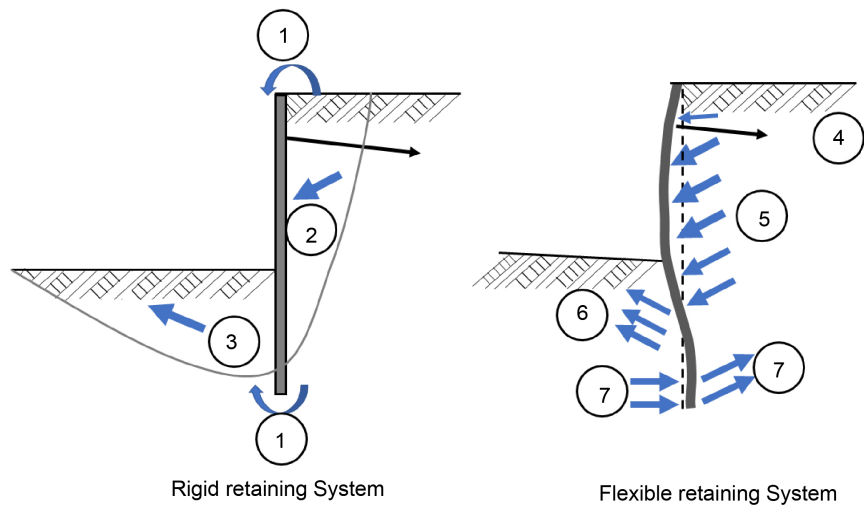


Figure 2. Kinematics of a Firm (Rigid) and Flexible Retaining Wall according to [5].

pressures exerted on the structure. For instance, in proximity to the support point (in this instance, the anchorage), deformations are marginal, contributing to a considerable mobilization of lateral pressure. Conversely, within the zone experiencing substantial deformations (typically the base of the excavation), deformations are markedly greater, yielding a diminished mobilization of lateral pressure.

In the present times, metal sheet piles are commonly utilized in marine or riverine environments (such as quays or river and canal banks) for their excellent waterproofing characteristics. They are also employed in various other configurations based on the project's requirements (near railways, in urban areas, etc.). Overall, opting for sheet piles offers an appealing cost-effectiveness ratio, largely due to the efficiency in execution owing to the industrialized production of these metal elements (manufactured in factories).

In terms of quality, the manufacturing process ensures a continuous and accessible quality control mechanism throughout the fabrication process. This stands in contrast to on-site constructions, where this process becomes intricate due to factors such as the site's nature, geological conditions, the nature of the structure, and the employed technological means. Nonetheless, the use of sheet pile retaining systems is not without disadvantages. Indeed, the installation process (implementation) through techniques like driving or sinking in rocky soils or firm sands presents considerable challenges and leads to significant noise disturbances in urban areas. Furthermore, transportation poses issues and restricts the extension beyond certain lengths, with a maximum of 24 meters per structure, as it is governed by road safety regulations and accessibility complexities.

Metal Sheet piling structures are composed of several metal elements assembled together to form a retaining screen. In 2003, Combarieu, *et al.* distinguished two categories:

- Single Panels: Irrespective of the existence or lack of supports, meaning whether the screen is cantilevered or supported by one or several lines of tie-backs or struts, the single panel consists solely of U-shaped (**Figure 3(a)**) or Z-shaped pieces (**Figure 3(b)** & **Figure 3(c)**). There are also flat profiles (linear), but these are rarely used. The choice of one profile over another can impact the structure's response. This is due to the location of the interlocking, which tends to reduce the structure's rigidity due to sliding between the metal elements. This phenomenon, occurring under bending loads, particularly concerns U-shaped profiles where the interlocking is located along the screen's axis, unlike Z-shaped profiles. To mitigate this, reinforcement at the interlocking level is achieved through clamping or welding techniques involving multiple elements (generally in pairs). Other factors also contribute to reducing this sliding between elements, such as friction at the interlocking level, friction at the interface, as well as horizontal reinforcements such as the cap beam (a reinforced concrete beam located at the topmost section of the screen) or tie beams (beams mainly used at the anchor point and occasionally elsewhere). From a normative standpoint, EUROCODES provide reducing safety factors to be applied to the moment of inertia of the structure and strength modules to better account for effects related to interlocking [7].
- Composite Panels: In this case, the panel is strengthened by vertical elements. These reinforcement elements are either composed of profiles identical to the structure (*i.e.*, opting for U-shaped or Z-shaped profiles to form caissons (**Figure 4**), or they are made of metal piles with tubular or H-shaped profiles (**Figure 4**). In this second configuration, the piles can also be used to support vertical loads (such as those from a crane or building). They then have greater inertia and depth in such cases.

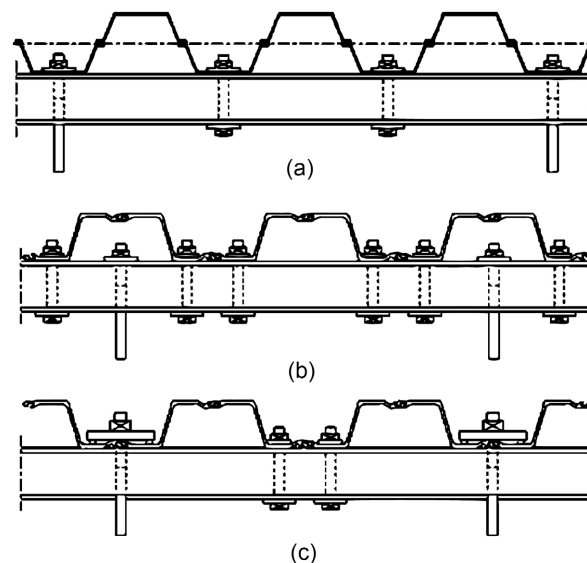


Figure 3. Plan view of the waler assemblies with sheet piles (a) U-shaped profiles, (b) and (c) Z-shaped profiles [6].

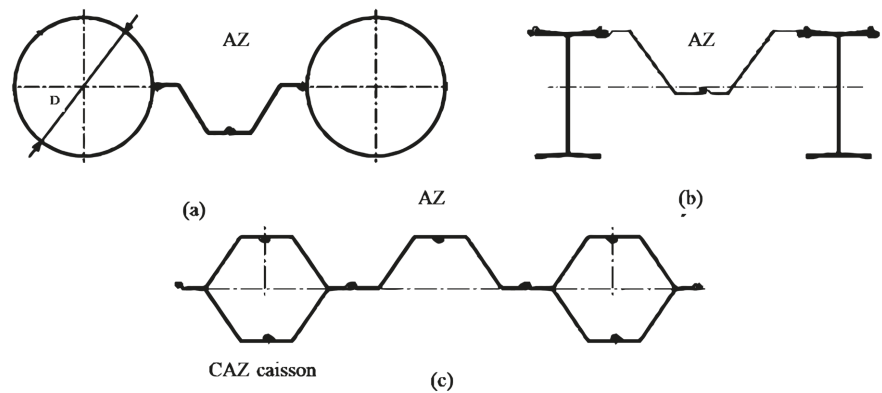


Figure 4. Plan view of mixed sheet pile wall assemblies: reinforcement using (a) tubular profiles, (b) H-shaped profiles, and (c) Z-shaped profiles adapted from [6].

According to Eurocode [7], there are four categories of sheet pile profiles, classified based on the steel's ductility, which refers to the material's ability to undergo plastic deformation without fracturing (Figure 5). This property is physically demonstrated in bending elements such as sheet piles, where it entails the capacity to rotate around a point.

Where: M_{pl} represents the plastic moment, M_{el} stands for the elastic moment, ϕ denotes the rotation, Φ_C signifies irreversible rotation associated with rotational capacity.

This categorization comprises the following classes:

- Class 1: characterized by the highest ductility (significant rotational capacity);
- Class 2: representing materials with low ductility;
- Class 3: indicating materials capable of reaching the elastic limit;
- Class 4: associated with materials strongly susceptible to buckling, warping, or tilting.

According to technical specifications provided by manufacturers in 2016 [9], it is evident that the majority of profiles fall within classes 2, 3, or 4. The sole means of achieving class 1 status would involve verifying the Capacity for rotation of class 2.

2.3. Failure Modes and Pathologies

Underestimation results in the achievement or surpassing of ultimate limit states (ULS) or serviceability limit states (SLS). This surpassing acts as the root cause of either complete or partial malfunctioning of the intended functionalities of the structure. Drawing insights from the research of Combarieu, *et al.*, 2003, this paper outlines the cluster of geomechanical parameters that could potentially trigger pathologies in a Sheet piling structure. The amalgamation illustrated in Table 2 brings to light the interconnectedness between several pathologies and the underlying triggers for their emergence. It's worth highlighting that the distinct structural components mentioned in Table 2 are visually identified in Fig-

Figure 6. A more comprehensive breakdown of various failure modes can be found in Eurocode 7 [10] or within the technical guide of Combarieu, *et al.*, 2003.

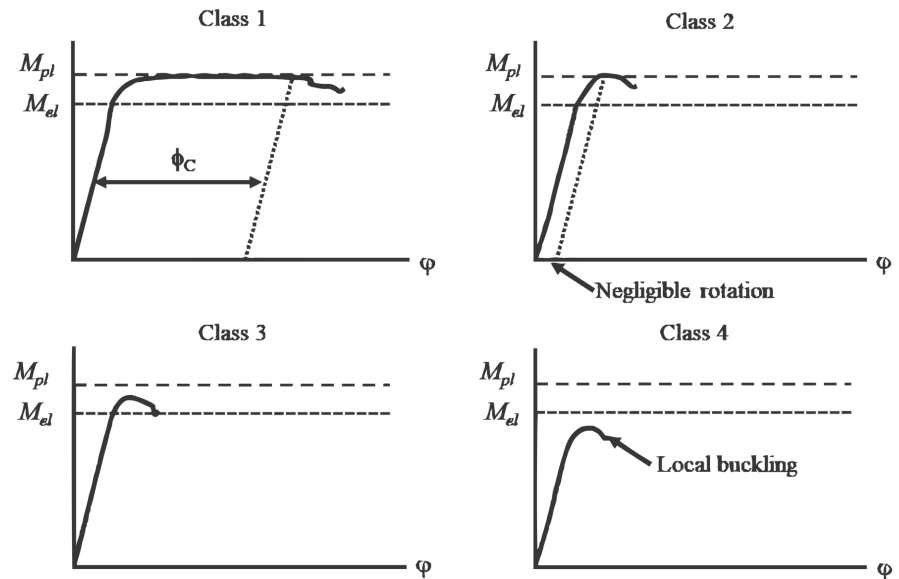


Figure 5. Steel Ductility Classes [8].

Table 2. Identification of Failures Impacting the Structure.

N°	Designation	Failure Origin	Potential pathologies
1	sheet pile	Undersizing of the sheet pile or premature refusal	Downstream slope at the top or bottom Planar deformation
2	Structure	Undersizing of the structure (inertia) Undersizing of the crossbeams Undersizing of the crown beam Excessive loading	Excessive deflection Cracking and Failure
3	Soil	Instability of the Anchor Massif Soil Heterogeneity	Significant displacement of the entirety Downstream Inclination of the Ensemble Planar Deformation
4	Tie rod	Undersizing of the tie rod (section) Inadequate spacing of tie rods Undersizing of the crossbeams Undersizing of the crown beam	Failure of the tie rod leading to structural instability
5	Strut	Same failure modes as for the tie rod	Failure of the struts leading to instability of the structure
6	Anchor block	Undersizing of the anchor element	Slippage of the tie rod leading to structural instability

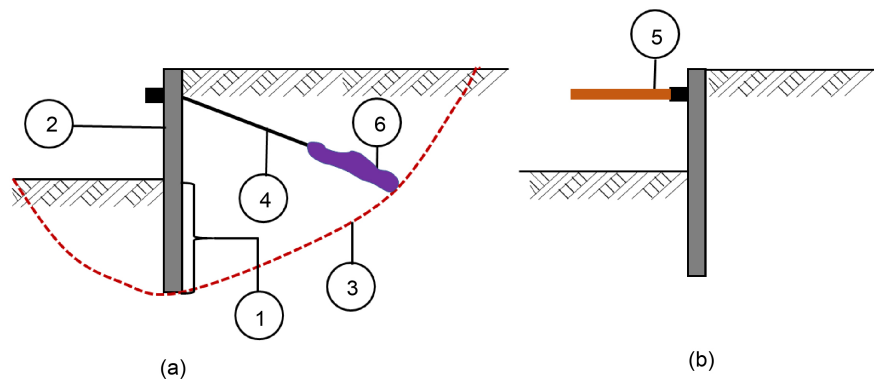


Figure 6. Identification of Failure Modes: Structure Supported by (a) Tieback and (b) Strut.

Upon studying **Table 2**, the fundamental role of the engineer, especially in the design phase, becomes evident. In fact, the under-dimensioning (U.D.) of one or more structural elements frequently paves the way for the appearance of pathologies, which can be ascribed to three types of reasons:

- Either the calculation methodology utilized for sizing this structural element isn't apt, thus failing to replicate its real-world behavior accurately;
- Or there has been a flawed assessment of external risk, which could also emanate from the partial safety factors utilized, falling short in encompassing uncertainties (random or epistemic);
- Or, most likely, a convergence of these two preceding factors. In actuality, the analysis of sheet pile walls structures frequently relies on linear sections and doesn't account for the spatial variability of the soil, resulting in a simplified mechanical model and a general handling of uncertainties.

Table 2 overlooks aspects tied to the degradation and aging of the structure. Notably, this form of pathology remains widespread, particularly in instances where sheet pile wall structures are situated in aquatic environments or aggressive soil conditions. The emergence of such pathologies gradually erodes the physical characteristics of the structure over the long term, leading to a diminution in mechanical strength and flexural rigidity.

3. Calculation Approaches for Sheet Piles

Two types of methods are commonly employed for the calculation and design of sheet piles [11]. These methods are categorized based on their consideration of soil-structure interaction (SSI).

The first category corresponds to the Limit Equilibrium Methods (LEM), which encompass analytical methods mainly intended for preliminary design, such as those proposed by Coulomb (1776), Rankine (1857), and Boussinesq (1876) [12] [13] [14]. For this category of methods, calculations are performed at the point of failure, assuming that the earth pressure is at the limit thrust upstream and the limit bearing downstream. In other words, the soil behavior is

considered to be consistently in the plastic phase, although in reality, this is not entirely accurate. Opting for LEM involves solving through a simple Strength of Materials analysis, employing force and bending moment equilibrium.

The second category comprises Soil-Structure Interaction Methods, referred to as SSIM. These are numerical methods based on a continuum representation using Finite Element (FE) or Finite Difference (FD) approaches. There's also a simplified approach based on the beam system and reaction coefficients (RC). SSIM methods are considered more accurate as they incorporate the geotechnical environment by accounting for soil-structure interaction (SSI). This article places particular emphasis on SSIM numerical methods.

3.1. Two-Dimensional Modeling Using SSI-SR

The adaptation of the beam model with elastic supports, also known as the Winkler method [15] (Equations (1)), for vertical structures (e.g. sheet piles, deep foundations) through the SSI-SR method (Figure 7) introduces several changes in the problem analysis. In this scenario, the soil is regarded both as a load source that varies with depth and as an encompassing medium interacting with the support. For continuous footings, the soil and the load remain entirely distinct; the soil takes on the role of the surrounding medium, while the load arises from an external application (e.g. superstructure). The forces exerted on a vertical structure hinge on the equilibrium state of the structure and are derived from elastic-plastic reaction laws (RL) for each depth level of the structure.

The equilibrium expression characterizing SSI-SRM can be concisely summarized as:

$$EI \frac{d^4 y(z)}{dz^4} + K_{ss}(z)y = 0 \quad (1)$$

where: EI represent the flexural rigidity of the beam, K_{ss} represent the soil reaction coefficient (representing ISS), y is the displacement of the structure, and z

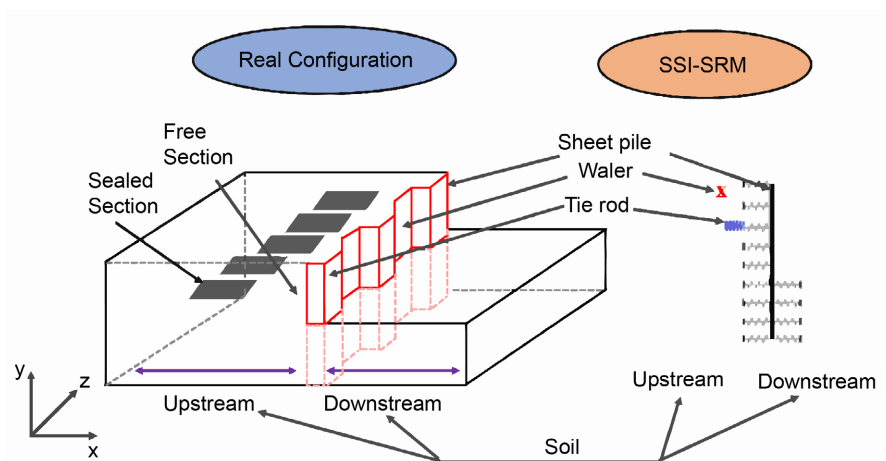


Figure 7. Illustration of the SSIM-SRM Method with Inclusion of the ISS, Transition from Real Configuration to a SSIM-SRM Representation.

the depth.

Various approaches can be employed to solve the problem, including:

- The analytical approach entails solving a second-order differential equation. This is accomplished by initially determining a specific solution and then acquiring the ultimate solution, as suggested by several commercially available software packages, such as K-Réa [16].
- The finite difference method [17];
- The transfer matrix method, Kort (2002);
- and the finite element method [18].

Regarding meshing, sheet pile wall structures are frequently depicted using beam elements, while soil-structure interaction and supports (tiebacks or struts) are modeled using springs. The concept of meshing in this type of methodology is less tangible, except for certain methods like FEM or FDM (finite element method or finite difference method, respectively), unlike purely analytical methods where meshing is unnecessary.

The adoption of the SR method relies in part on the utilization of reaction laws (RL) to replicate the overall behavior of SSI in a highly simplified manner while also integrating the stiffness of the structure and the soil, along with the plastic limits of the latter. According to Kramer, 1988, the fundamental concept of this approach is to establish a connection between the mobilized forces and the movement of the structure, known as the P - Y relationship. Just like classical constitutive models (e.g. Mohr-Coulomb, Cam-Clay), reaction laws also necessitate the definition of a failure criterion to bound the elastic range. These boundaries are determined by the rupture mechanisms that govern the soil surrounding the support [19].

Within the literature, there are several types of RL tailored for specific scenarios with distinct failure criteria. These RL often hinge on factors such as soil type (frictional or cohesive), its stiffness (stiff or soft soil), the presence of the water table, and sometimes the rigidity of the structure itself. For example, Matlock (1970) put forward a model for soft clays submerged beneath a water table [20]. Reese and Welch (1975) introduced a model quite similar to Matlock's (1970), adapted for flexible piles in stiff clays without a water table. Other studies such as, Reese, *et al.* (1974), introduced a criterion suitable for stiff clays in the presence of a water table [21]. Finally, Sullivan, *et al.* (1980) presented a generalized criterion applicable to all submerged clays [22].

In practice, managing this variety of situations is complex and necessitates adjusting the reaction law to match the imposed conditions. Another viable option is employing the Mohr-Coulomb failure criterion (Figure 8). For Magnan & Mestat (1997), the benefit of this criterion is its ability to replicate various soil types under diverse hydraulic conditions [23].

The Equations (2), (3), and (4) summarizing this criterion are:

$$\sigma'_a = \sigma'_v K_a - 2c' \sqrt{K_a} \quad (2)$$

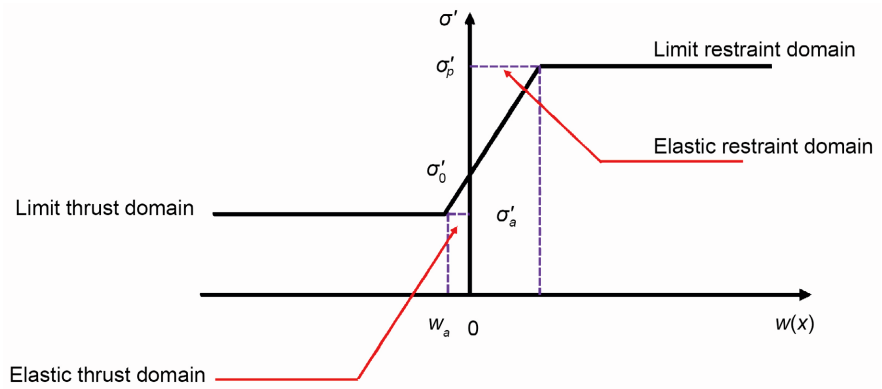


Figure 8. Reaction law for a calculation segment.

$$\sigma'_p = \sigma'_v K_p - 2c' \sqrt{K_p} \tag{3}$$

$$\sigma'_0 = K_0 n_v \tag{4}$$

where: σ'_a and σ'_p respectively stand for effective thrust and bearing stresses, σ'_v signifies the vertical stress of the soil, c' represents the cohesion of the soil. It is noteworthy that the assessment is conducted using effective stress, which implies the subtraction of water weight from the soil weight (buoyant unit weight), and the hydrostatic pressure is applied directly to the wall. K_0 represents the coefficient of at-rest earth pressure.

As for K_a and K_p , they correspond to the coefficients of thrust and bearing of the soil, which can be determined using various existing analytical expressions such as Coulomb (1776); Rankine (1857); Muller-Breslau (1906) and Kerisel & Absi (1990) [24] [25]. These coefficients primarily rely on the soil's friction angle, wall roughness, and the geometry of the supported soil surface (horizontal or inclined).

The expressions for K_a and K_p for an upright wall and zero inclination of the upstream slope are derived from Rankine (1857) (Equations (5) & (6)):

$$K_a = \tan^2 \left(\frac{\pi}{4} - \frac{\phi}{2} \right) \tag{5}$$

$$K_p = \tan^2 \left(\frac{\pi}{4} + \frac{\phi}{2} \right) \tag{6}$$

The expressions of K_a and K_p for an upright wall and zero upstream slope inclination are written as per Coulomb (1776) (Equations (7) & (8)):

$$K_a = \frac{\cos^2 \phi}{\cos \delta \left[1 + \sqrt{\frac{\sin(\phi + \delta) \sin \phi}{\cos \delta}} \right]^2} \tag{7}$$

$$K_p = \frac{\cos^2 \phi}{\cos \delta \left[1 - \sqrt{\frac{\sin(\phi + \delta) \sin \phi}{\cos \delta}} \right]^2} \tag{8}$$

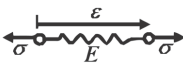
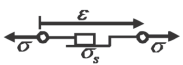
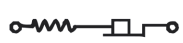
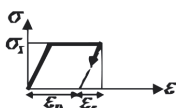
From a rheological perspective (**Table 3**), a perfectly elastic-plastic behavior is depicted by serially connecting a spring-like element (representing elastic behavior) with a pad element (representing the plastic zone). The elastic phase comes into play only when the pad is activated, i.e. when the defined threshold values are not exceeded.

The accurate determination of elastic stiffness used in the SSI-SR method significantly contributes to its effectiveness. K_{ss} serves two primary purposes: firstly, it characterizes the stiffness of the soil-structure system at a specific point on the structure (either rigid or flexible); secondly, it defines the plasticity thresholds, wherein higher K_{ss} stiffness leads to a smaller elastic range, and vice versa. The evaluation of K_{ss} relies on empirical expressions, which may not always have straightforward mechanical explanations. These expressions often stem from practical experience and on-site observations. To date, several expressions are available for this calculation. A comparison conducted by Kazmierczak (1996) highlights the substantial variation in K_{ss} values depending on the method used [26]. Most of these methods are partially based on the soil’s compressibility module (pressiometric module), with the exception of Chadeisson’s charts, according to Monnet (1994), which rely more on soil strength characteristics (cohesion and friction angle) [27]. In the following, a brief overview is provided on a few methods to estimate K_{ss} .

The method known as the Ménard and Balay method was initially proposed in 1965 by Ménard & Bourdon (Equations (9)) [28]. The original expression only described the stiffness at the pile level. Subsequent versions were introduced to provide a description across the entire height of the structure, notably by Balay in 1984 [29] and later by Josseume *et al.* (1997) [30]. These adjustments primarily involve the geometric parameter A, determined for construction stages different from those in the initial version. For instance, these adaptations accommodate variations in pile dimensions or configurations of excavated zones, or to incorporate the installation of supports (tiebacks or struts).

$$K_{ss} = \frac{E_M}{0.5\alpha A + 0.133(9A)^\alpha} \tag{9}$$

Table 3. Review of Rheological Models.

	Models	Behavioral Law	Stress
Spring		Linear Elastic	$\sigma = E\varepsilon$
Pad		Perfect Plasticity	$ \sigma \leq \sigma_s$
Series Connection		Perfect elastoplastic behavior	

where:

- E_M is pressiometric modulus of the soil in the considered layer;
- A : dimensional parameter determined according to Josseume, *et al.* (1997) as displayed in **Figure 9**;
- α rheological coefficient of the soil (**Table 4**) depending on the soil type and consolidation type (e.g., 1/3 for normally consolidated sandy soils).

In 1995, Schmitt introduced a method based on a series of measurements conducted on actual structures [31]. This approach incorporates the mechanical features of the structure into the calculation by means of its flexural stiffness (Equations (10)). It could potentially be employed within the context of the standard (NF P94-282, 2009) [32] after undergoing additional verification steps. However, It's noteworthy that a significant shortcoming of this approach is its failure to account for excavation geometry.

$$K_{SS} = 2.1 \frac{\left(\frac{E_M}{\alpha}\right)^{\frac{4}{3}}}{\left(\frac{E_p I_p}{B}\right)^{\frac{1}{3}}} \tag{10}$$

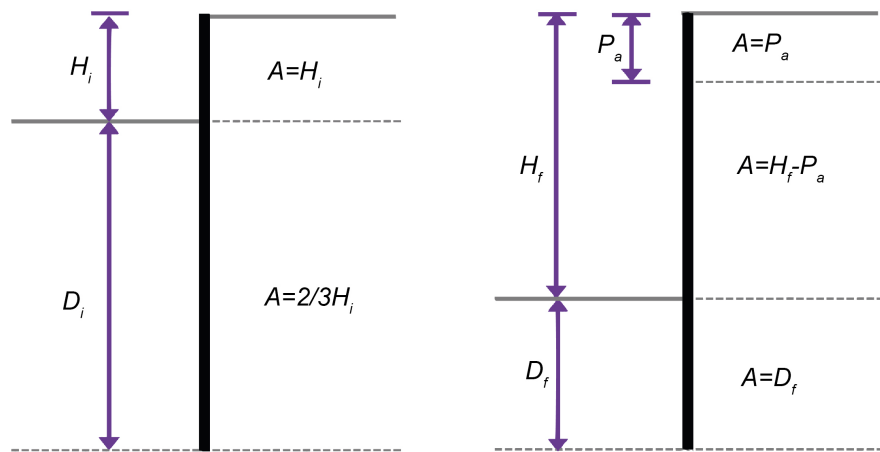


Figure 9. Calculation of the dimensional parameter A, as per Josseume *et al.* (1997) [30].

Table 4. Rheological coefficients α based on geotechnical criteria (NF P94-282, 2009).

Type	Peat		Clay		Silt		Sand		Gravel	
	E_M/pl	α	E_M/pl	α	E_M/pl	α	E_M/pl	α	E_M/pl	α
Overconsolidated or very compacted	--	--	>16	1	>14	2/3	>14	1/2	>14	1/3
Normally consolidated or normally compacted	--	1	9-16	2/3	8-14	1/2	8-14	1/3	8-14	1/4
Altered and reworked underconsolidated or loose	--	--	7-9	1/2	5-8	1/2	5-8	1/3	--	--

where: E_M is Ménard's pressiometric modulus, $E_p I_p$ is the flexural stiffness of the sheet pile, B is the reference length set at 1 m, and α is rheological coefficient of the soil (**Table 4**).

The representation of support elements like tiebacks or struts is accomplished through the utilization of elastic spring elements. Determining the stiffness of such a support follows Equation (11) (Balay, 1988), treating it as a component under tension or compression, utilizing the term $E_a S_a / L_{eff}$ where L_{eff} signifies the effective length of the element, E_a denotes its Young's modulus, and S_a represents its cross-sectional area [33]. It's important to note that L_{eff} corresponds to the actual length of a strut-type support (L_{but}). For a tieback, it involves the summation of the untensioned length (L_{lib} ; free length of the tieback) and half of the embedment length (L_{scel}) (**Table 5**). In scenarios where the support is inclined relative to the horizontal, the expression is adjusted by $\cos^2 \alpha$, where α indicates the inclination angle of the support concerning the horizontal.

$$K_T = \frac{E_a S_a \cos^2 \alpha}{L_{eff}} \quad (11)$$

Much like in soil-structure interaction modeling, constraining the elastic phase is also achievable by introducing a plasticity threshold, whether with or without isotropic hardening. This threshold can correspond to the ultimate state of the steel or the occurrence of slipping (disengagement) within the anchorage. Establishing the utmost stress of the steel, e.g., Strom & Ebeling, (2001) is comparatively straightforward, as opposed to determining the sliding threshold of the anchoring material, which partly relies on soil characteristics and anchoring material properties [18]. In the latter case, according to Bustamante & Doix (1985), the calculation involves estimating the load that can be mobilized through friction, similar to methods used for micropiles [34].

3.2. Two-Dimensional Modeling Using SSI-FE

The principle of FEM (Finite Element Method) involves finding a numerical solution for a problem described by partial differential equations within a finite domain. The SSI-FE (Soil-Structure Interaction Finite Element) method is an advanced computational technique widely utilized in civil engineering. Numerous researchers, including Mokeddem *et al.* [35] [36] and Boussinesq (1876) have employed this principle to gain insights into the behavior of sheet pile wall structures while accounting for the spatial variability of the ground. Its primary aim is to investigate the complex interaction among structures and the surrounding soil, enabling engineers to develop a deeper understanding of how

Table 5. Effective Length in accordance with the category of Support.

Support	L_{eff}
Strut	L_{but}
Tie rod	$L_{lib} + 0.5L_{scel}$

these elements interact under various scenarios.

At its core, the SSI-FE method combines two fundamental approaches: the Finite Element (FE) analysis technique and the consideration of Soil-Structure Interaction (SSI). This fusion enables engineers to create a simulated depiction of both the structure and the ground in discrete segments, effectively simulating their behavior under different conditions.

Through iterative mathematical computations, the SSI-FE method predicts how external forces, loads, and displacements impact both the structure and the ground. This dynamic assessment considers the mutual influence between the two, resulting in a more accurate and realistic depiction of their behavior when subjected to real-world conditions.

The SSI-FE method finds significant application in scenarios where the interaction between structures and soil plays a pivotal role. Examples encompass the evaluation of foundation systems, the behavior of retaining walls, and the response of subterranean structures. By integrating the SSI-FE method into engineering analyses, professionals can make well-informed decisions regarding design, construction, and safety measures.

Essentially, the SSI-FE method serves as an advanced tool for exploring the intricate relationship between structures and soil. It enhances the precision of engineering assessments, facilitating the creation of robust solutions capable of withstanding complex environmental factors and loading conditions. In the context of this paper, employing a solid mass representation for modeling a retaining wall provides a more accurate approximation of the real response of the structure within its environment, allowing for a better consideration of soil-structure interaction.

In practice, modeling a retaining wall using this method in continuum representation (SSIM-FEM) provides a more accurate approximation of the real response of the structure within its environment, achieving improved incorporation of Soil-Structure Interaction (SSI), more rigorous soil modeling, and refined interface representations. However, practical engineering often lacks complete mastery and understanding of certain calculation aspects, such as soil behavior laws, leading to ongoing questions about choices and assumptions to be made.

Figure 10 encompasses all the elements typically employed in the framework of modeling using SSIM-FEM method.

The calculation process through SSI-FEM is nearly analogous to that of any other finite element analysis, with a few distinct details. Primarily, these pertain to loading arising from thrust or bearing forces mobilized due to the relocation of the structure. These steps are primarily summarized in Ou (2006) [37] [38] and Smith, *et al.* (2013) [39].

In conditions of planar stress and for an isotropic material, the stress-strain relationship is formulated in an expanded manner (Equations (12)) and in a compact form (Equation (13)).

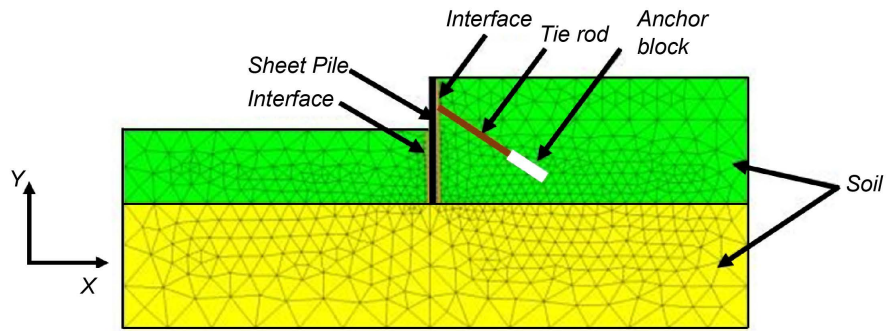


Figure 10. Schematic of the SSI-FE Method Principle.

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \cdot \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (12)$$

$$[\sigma] = [D] \cdot [\varepsilon] \quad (13)$$

where: σ_x , σ_y , and τ_{xy} represent the stress values in the x-direction, y-axis, and in-plane stress, respectively. x , y , and γ_{xy} represent the strains along the x-axis, y-axis, and in-plane strain, respectively. Furthermore, the displacement-strain relationship can be described by Equation (14):

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \cdot \begin{bmatrix} u \\ v \end{bmatrix} \quad (14)$$

To simplify Equation 3, it can be expressed in the following form (Equation (15)):

$$[\varepsilon] = [d] \cdot [U] \quad (15)$$

where:

- u and v represent the displacements along the x and y directions of the mesh element.
- ε : the vector of strains.
- d : the matrix of partial derivatives.
- U : the vector of displacements, with u representing the horizontal displacement and v representing the vertical displacement.

The change in position at a specific location of the finite element can be expressed as a function of the nodal displacements q (Figure 11) using shape functions $[f]$, which change based on the mesh configuration.

$$[U] = [f] \cdot [q] \quad (16)$$

With:

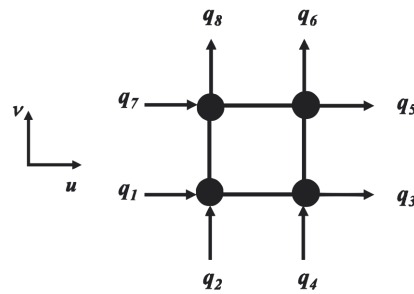


Figure 11. Illustration of Nodal Displacements.

$$[U] = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_8 \end{bmatrix} \quad (17)$$

By substituting Equation (16) into Equation (15), Equation (18) is derived:

$$[\varepsilon] = [d] \cdot [f] \cdot [q] \quad (18)$$

which can be expressed in the form of Equation (19):

$$[\varepsilon] = [B] \cdot [q] \quad (19)$$

By applying the principle of virtual work, the elemental stiffness matrix $[k_e]$ of a finite element can be obtained (Equation (20)):

$$[k_e] = \iint [B]^T [D] [B] dx dy \quad (20)$$

After constructing all the elemental matrices, they are combined into a global matrix denoted as $[K]$. The finite element analysis can be represented by the Equation (21):

$$[K][q] = [P] \quad (21)$$

$[P]$: represents nodal forces resulting from soil excavation (or embankment). In addition, external loads can be applied either directly or transferred (through the soil) to the structural nodes (e.g. operational loads).

The choice of meshing depends on the problem's dimensions and the nature of the element being modeled. Several researchers have investigated on meshing selection, including Arafati (1996) [40]; Mestat P. and Arafati N. (1998) [41]; Vossoughi (2001) [42]; Nguyen (2003) [43]; Mestat, *et al.* (2004) [44] [45]. The synthesis presented in **Table 6** is established, on one hand, from these researchers' work and, on the other hand, from reflections on the operational mode and roles of certain structural elements. The main meshing elements, their density, and the boundary conditions adopted to describe the components of a sheet pile wall and its environment are outlined. For instance, to model the 2D soil mass, the choice often falls on triangular or quadrilateral elements. The mesh density depends on the desired accuracy; finer meshing enhances calculation precision but also increases computational time costs. Vossoughi, 2001, recommended employing variable density, opting for coarser meshing in distant zones and

Table 6. Comprehensive overview of meshing components in the assessment of a sheet pile retaining structure utilizing the MISS-EF method [40] [41] [42] [43] [44].

Element	Meshing Element	Geometric Mesh Element		Density	Boundary conditions
		2D	3D		
Soil	Bedrock	X	X	Fine Meshing Towards the Structure	Single supports on the right and left; Articulation at the bottom edge
Tie Rod	Bar	X	X	Single element	Articulations at the junction of tie rod and sheet pile
	Spring	X	X	/	
Anchor Block	Bar	X	X	Uniform density	The points within the mass are connected to the soil
	spring	X	X	/	The spring represents the boundary condition
Abutment	Bar	X	X	Single element	Articulation
	Beam	X	X	Uniform density	Articulation
	Imposed Displacement	X	X	/	Considered as restrained
	Solid	X	X	Uniform density	at a specified distance Articulation
Crossbeam	Beam	/	X	Uniform density	Linear support along the sheet pile
Interface	Joint or interface elements	X	X	Same density as the adjacent soil	This element represents the boundary condition

finer meshing in zones near the structure [42]. Lastly, it's crucial to ensure that boundary conditions don't affect the structure's behavior through unwanted effects, achieved by selecting appropriate dimensions (length and depth of the mass).

Regarding the sheet pile modeling, it can be done using beam elements or continuum elements. According to Mestat, *et al.*, 2004, a database informed by various research has found that 3/5 of studies opt for this representation. The same observation applies to current commercially used software like PLAXIS [46], and FLAC [47] [48], which also favor this approach. In terms of accuracy, differences can exist between the two approaches (beam and continuum). According to Arafati, 1996, theoretically, beam elements are likely better suited for sheet pile walls, while continuum elements are more appropriate for stiffer screens.

Several constitutive models (CM) are available to replicate the behavior of various types of soils, including rocks, granular soils (highly permeable granular soils), or cohesive soils (fine soils with low permeability). These CMs rely on geotechnical characteristics determined through laboratory tests such as triaxial and oedometer tests, as well as in-situ tests like the pressuremeter test. Hydraulic conditions (drained and undrained) are also vital considerations during these tests.

A classification into two categories of soil CMs was proposed by Magnan &

Mestat (1997):

- “Simple” behavior models: This category encompasses the linear elastic model. Note that the application domain is limited to lightly stressed soils with deformations not exceeding 0.01% or less, as demonstrated by Tatsuoka & Shibuya (1991) [49], cited by Doan Tran (2006) [50]. Other CMs also fall into this category by integrating plasticity thresholds, such as the Mohr-Coulomb or Drucker-Prager models, to confine the elastic domain.
- “Advanced” behavior models: Examples include incremental and nonlinear models like elastoplastic models with hardening (e.g., Cam-Clay), models incorporating nonlinearity in the elastic phase (e.g., Hujeux), and more. The purpose of these models is to express specific behaviors, such as small deformations by incorporating nonlinearity in the elastic phase, as seen in the Hujeux model, for instance.

The utilization of a “simple” CM remains more common nowadays compared to that of an “advanced” CM. According to Kort, (2002) this can be attributed firstly to the challenge of determining certain non-physical parameters for “advanced” behavior models, and secondly, due to the sometimes greater resolution time. For the modeling of support systems and excavations, simplified elastoplastic models like Mohr-Coulomb are often selected. Delattre (2004) exposed that the modeling parameters for such models are determined through common tests (triaxial and pressuremeter) [51]. This trend can be observed through the MOMIS database (2004), which compiles various numerical studies from literature focusing on support cases. This database recorded around 48% of non-hardening models (like Mohr-Coulomb). Alternatively, 23% of the studies employed hardening-integrated methods (like Cam-Clay), and finally, 29% relied on purely elastic models. The performance of the Mohr-Coulomb model remains highly significant, as indicated by comparisons conducted by Kort (2002) within the context of the Rotterdam benchmark [52].

Given the different stiffnesses of the structure and the adjacent soil, a differential displacement between the two elements can arise, leading to sliding and/or detachment. This directly affects the surface of the soil located upstream of the structure (behind the structure) and results in soil settlement. Neglecting the interface can lead to an unrealistic uplift of the soil upstream.

From a numerical standpoint, Day, 1994 argues that modeling the interface (**Figure 12**) must be carefully considered to avoid causing non-convergence in calculations [53]. This modeling depends on several parameters (geometric, mechanical, and geotechnical) that can be integrated into different approaches: continuous finite elements, spring connections, joint elements, etc. Potts often opts for joint elements (thin or zero-thickness elements), which seem better suited for geotechnical issues [54]. Indeed, the work conducted by Elmi, *et al.* (2006) [55] in the context of modeling the instrumented Hochstetten structure [56] demonstrates the advantages of this joint model, such as its straightforward approach, robustness, and rapid convergence compared to non-zero-thickness

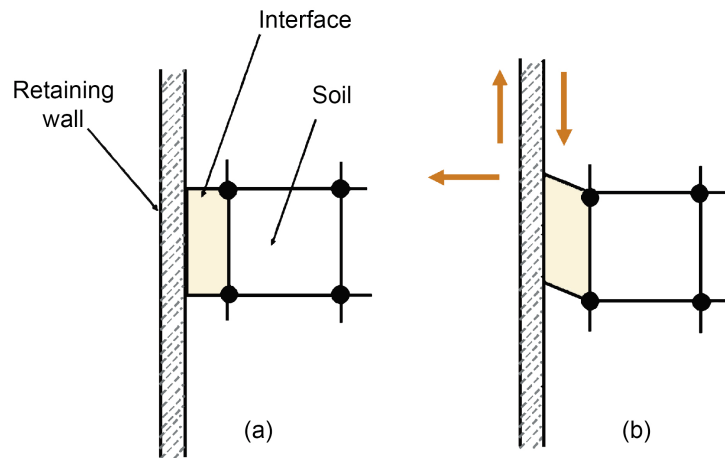


Figure 12. (a) Finite element representation of the interface (thin layer), (b) Kinematics of elements: soil-interface-structure

elements. The joint model enables the expression of stresses (normal $\Delta\sigma$ and tangential $\Delta\tau$) as functions of strains (respectively, normal and tangential for $\Delta\varepsilon_n$ and $\Delta\varepsilon_t$) (Equation (22)) and describes stresses in the two-dimensional elastic domain.

$$\begin{bmatrix} \Delta\sigma \\ \Delta\tau \end{bmatrix} = \begin{bmatrix} K_n & 0 \\ 0 & K_t \end{bmatrix} \begin{bmatrix} \Delta\varepsilon_n \\ \Delta\varepsilon_t \end{bmatrix} \quad (22)$$

In this expression, K_n and K_t respectively represent the perpendicular and tangential elastic rigidities.

Day and Potts suggest that in theory, determining interface stiffness requires a laboratory test campaign such as a shear box test [57]. However, in practice, according to Beer, this is rarely the case, and analytical expressions are often preferred, such as those proposed by Beer (1985) (Equations (23), (24)) [58].

$$K_n = \frac{E_s}{ep_j} \quad (23)$$

$$K_t = \frac{E_s}{2(1+\nu_s)ep_j} \quad (24)$$

where:

- E_s represents the Young's modulus of the soil;
- ν_s represents the Poisson's ratio of the soil;
- e_{pj} represents the thickness of the joint.

According to Arafati, 1996, based on physical considerations, the thickness e_{pj} is a fictitious dimension and thus not measurable. However, its value must be carefully chosen as it significantly influences the order of magnitude of K_n and K_t . Moreover, it is often recommended in the literature to opt for high values of K_n to avoid any issue of interpenetration between the two contacting elements. This leads, as noted by Benhamida and Nguyen, to relatively small values of e_{pj} compared to the distance between two successive nodes of the same

element (soil or structure), while avoiding nearly zero values that lead to numerical singularities [59]. There is no specific standard to define this distance precisely; nonetheless, the values typically used for this magnitude are on the order of centimeters, with Day & Potts proposing a thickness of 2 cm.

For retaining wall problems, soil and interface plasticization is quickly reached, highlighting the importance of limiting the plastic zone to describe sliding and detachment (Table 7). The Mohr-Coulomb criterion (Figure 11) is often employed in this context to restrict the shear stress. Once the limiting stress is reached, sliding occurs between the two surfaces (the shear stiffness becomes zero while the normal stiffness remains unchanged).

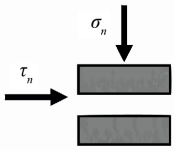
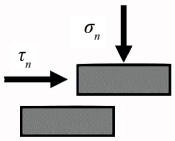
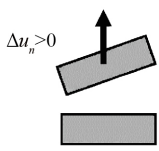
Furthermore, Arafati argues that when tension occurs in the interface, both stiffnesses become zero, resulting in a redistribution of shear stress (Table 7). Generally, this model relying on the Mohr-Coulomb failure criterion appears quite satisfactory for representing the interfaces of retaining structures, as demonstrated in the study by Day & Potts (1998) [57], which compared the results obtained using this approach with those of the classical Caquot-Kerisel (1948) method [60]. However, as Dong (2014) points out, numerical complications sometimes arise due to this criterion (Equation (25)) [61].

$$F = |\tau| + \sigma' \cdot \tan \delta - c' \tag{25}$$

where:

- τ is the tangential stress;
- c' represents the material's cohesion;
- σ' indicates the effective normal stress applied on the plane;
- δ denotes the angle of inner friction between soil and wall (maximum shear angle).

Table 7. Interaction Modes of Interfaces according to Prat, *et al.* (1995) [62].

Interaction Mode	Representation	Displacement	Stress
Adhesion		$\Delta u_n = 0$ $\Delta u_t = 0$	$\sigma' > 0$ $\tau < C_a + \sigma'_n \tan \delta$
Sliding		$\Delta u_n = 0$ $\Delta u_t \neq 0$	$\sigma' > 0$ $\tau = C_a + \sigma'_n \tan \delta$
Detachment		$\Delta u_n > 0$ $\Delta u_t \neq 0$	$\sigma' = 0$ $\tau = 0$

The parameter δ actually corresponds to the product of a ratio R by the angle of inner friction of the soil ϕ' $\delta = R \cdot \phi'$. It serves to characterize the effect of the roughness of the soil-structure interface. This ratio depends on the nature of the materials constituting the retaining wall and the surrounding soil. In practice, sheet pile walls are regarded as having rough surfaces (untreated steel), and determining the value of R can be subject to uncertainty. In fact, even within French literature, there are discrepancies:

- The work of Prat, *et al.* (1995), which is more general in its approach to modeling civil engineering structures, provides intervals for describing δ directly based on the nature of the soil in a very broad manner [62].
- The work of Mestat, *et al.* (1999), which remains a strong reference in the numerical analysis of geotechnical structures, simply uses the value $R = 2/3$ without specific details. The work of Mestat, *et al.* (1999), which remains a strong reference in the numerical analysis of geotechnical structures, simply uses the value $R = 2/3$ without specific details [63].
- Lastly, the work of (Philipponat, 2016), also widely used by French geotechnical consulting firms (including its earlier editions), considers that the value of R depends on the angle of internal friction's numeric representation of the soil ϕ and the orientation of the earth pressures (push or passive) [64] [65].

The soil-anchor interaction partly depends on the nature of anchor used and the nature of the anchoring mass. In practice, most commercially used finite element analysis software (e.g. (PLAXIS, 2016)) rely on a simplified modeling approach, considering this interaction only partially by modeling it solely at the level of the anchoring mass, assuming that the latter is perfectly integral with the soil (Figure 13). This implies that no sliding is possible between the two. A more comprehensive consideration of this behavior would complicate matters, as it would require describing the problem in a three-dimensional scale with non-linear behavior, as shown by Desai, *et al.* (1986) [66]. Additionally, it would be necessary to incorporate the coupling effect in the presence of water.

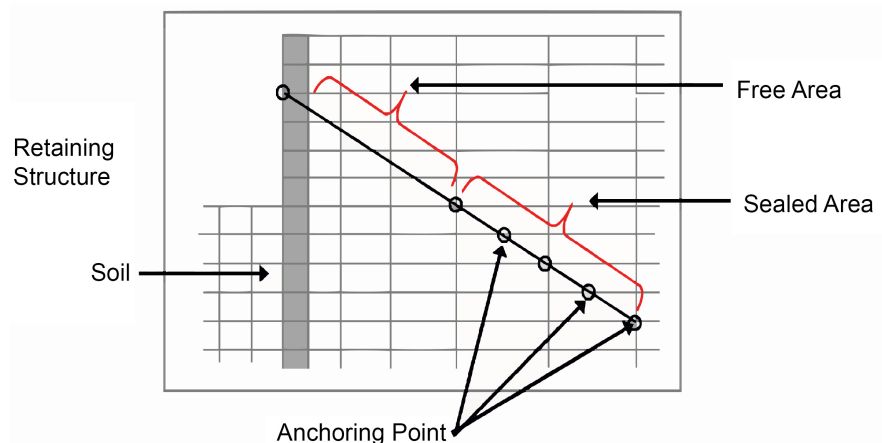


Figure 13. Interconnection between anchor block and surrounding soil [65].

3.3. Three-Dimensional Calculation of Sheet Piles—Eurocode 7 and Methods for Sheet Pile Wall Analysis

Despite the significant strides made in computational technology over the last few years, the three-dimensional analysis of geotechnical structures has remained relatively limited in practical application. This is particularly evident in cases involving retaining structures, where the longitudinal dimension is seldom incorporated into the analysis. Generally speaking (across all categories of retaining walls), three-dimensional analysis is reserved for exceptional scenarios, notably in urban settings where excavation and resultant displacements can exert an influence on existing buildings. In this context, the work by Chehade, *et al.* (2010) has underscored the utility of employing a 3D model, especially when dealing with point loads (isolated footings) adjacent to a retaining structure [67]. This analytical approach is also frequently utilized for deep excavations necessitating the construction of multiple underground levels, as highlighted by Popa, *et al.* (2009) [68], wherein temporary strut phases are succeeded by permanent floor-supported phases. Kort (2002) conducted a contrastive study of 2D and 3D models for a structure instrumented in Rotterdam. The study revealed that stress-based models employing plane stress assumptions (2D) tended to overestimate bending moments and displacements. This could be ascribed to the omission of certain structural elements, particularly at the structure's periphery.

Due to the intricacy and computational demands of numerical models, simplified approaches have gained prominence. For instance, Finno, *et al.* (2007) proposed a method that involved a parametric investigation of 3D structures, resulting in the derivation of a displacement correction parameter at the structure's center, denoted as PSR (Plane Strain Ratio) [69]. This semi-empirical approach hinges on displacement values obtained through a cross-sectional analysis (2D) of the structure.

In a similar vein of simplification, Bryson & Zapata-Medina (2012) also introduced a semi-empirical method [70]. This methodology allows for the examination of system stiffness by amalgamating three-dimensional analysis and incorporating soil and structural characteristics. Their study encompassed various wall types and culminated in the establishment of stiffness ratios employed to prognosticate structural behavior.

In general, the investigation of sheet pile walls is governed by the European standard applied to geotechnical structures: Eurocode 7 NF EN 1997-1 (2005), along with the French annex NF P94-282 (2009). Upon reviewing this standard, a distinct hierarchy emerges in the selection of calculation methods for retaining structures. Specifically, initial design is established through employment of MEL methods, which enable assessment of minimal sheet pile dimensions and earth pressure for straightforward structures NF P94-282 (2009). The SSIM-SRM method, however, is preferred to delineate the structure's behavior, particularly its displacement. In contrast, the utilization of the SSIM-FEM (or DF) method is restricted to intricate projects featuring complex geometries, interplay between

structures, and comparable circumstances.

Unlike the LEM methods and the SSIM-SR approach, the standard NF P94-282 (2009) appears somewhat cautious in its endorsement of MISS-EF. Recommendations for its application remain relatively high-level. For instance, specific directives regarding the application of behavioral laws or the modeling of the soil-structure interface are absent.

Regarding the “dimension” of the calculation to be adopted, computations are conventionally conducted in cross-sectional fashion (2D). The adoption of three-dimensional calculations is generally deemed unnecessary, given the assumption of soil homogeneity. As of now, various assumptions remain the prerogative of the geotechnical engineer, encompassing the choice of calculation methodology, behavior or reaction laws, and more.

3.4. Comparison of Approaches for Sheet Pile Wall Analysis

This paper presents two types of comparisons among various retaining wall calculation methods. The first comparison primarily addresses the geomechanical aspects of the soil-structure interaction. The second comparison involves assessing these methods based on criteria related to effectiveness and efficiency. The goal of this segment is to discern the merits and drawbacks of each method and subsequently categorize them according to specific requirements. For the purpose of clarity in notation, the SSIM method represented as FE (finite elements) and FD (finite differences) is collectively referred to as SSIM-FEM.

The transition from the physical realm to the numerical domain introduces errors and uncertainties. The choice of method and the assumptions made beforehand (input data) ultimately falls to the designer. This leads to varying performances (the ability to replicate physical behavior) and sensitivities. Theoretical comparison among different methods for calculating retaining walls (**Table 8**) based on a range of geomechanical criteria reveals that the SSI-FEM method is the most capable of rigorously replicating behavior. For instance, representing the soil with solid elements yields more accurate outcomes for soil behavior and deformations (settlement). Additionally, the SSI-SR method is likely less strict than the former, but strikes a balance between precision (taking into account ISS) and computation time (reduced time). Finally, the LEM method remains a highly simplified approach primarily used for preliminary design, assuming that soil behavior is consistently in the plastic phase.

Sometimes, better performance can be achieved by relying on a simplified approach, such as the SR method. Indeed, within the context of the competition organized to predict the response of the Rotterdam excavation, it was observed that SR calculations sometimes provided more consistent results with in-situ measurements (e.g., structure displacement). This can be partly explained, as indicated by Kort (2002), by the challenge associated with determining certain non-physical parameters of the behavior laws used in the SSI-FE method.

To evaluate these different methods in terms of practical application, three

Table 8. Comparison of sheet pile assessment methods based on geometric and mechanical criteria.

Criteria	LEM	SSI-SRM	SSI-FEM
Dimension	2D	2D	2D & 3D
Modeling of soil, ISS, and soil-wall inter-face	<ul style="list-style-type: none"> - Absence of soil modeling, direct utilization of earth pressure and limit constraints - Absence of Interaction Soil Structures (ISS) - Incorporation of interface roughness 	<ul style="list-style-type: none"> - No soil modeling - Simplified modeling of Interaction with Soil Structures (ISS) with the possibility of different reaction laws- - Consideration of the Retaining Wall's roughness (interface) 	<ul style="list-style-type: none"> - Rigorous soil modeling with the possibility of different behavior laws - Consideration of Interaction with Soil Structures (ISS) - Interface modeling
Modeling of the sheet pile structure	<ul style="list-style-type: none"> - Modeled as a beam - Suited for rigid Retaining Wall 	<ul style="list-style-type: none"> - Beam - Rigid and Flexible Retaining Wall 	<ul style="list-style-type: none"> - Beam or solid element - Rigid and Flexible Retaining Wall
Modeling of tie rods/buttrresses	<ul style="list-style-type: none"> - Simple supports - Only one layer of tie or strut allowed 	<ul style="list-style-type: none"> - Elastic Spring - Multiple layers of tie or strut allowed 	<ul style="list-style-type: none"> - Bar element with boundary conditions at the anchor block - Multiple bearing layers allowed
Incorporation of phasing	<ul style="list-style-type: none"> - Not appropriate for this analysis 	<ul style="list-style-type: none"> - Possibility of integration 	<ul style="list-style-type: none"> - Possibility of integration
Computation	<ul style="list-style-type: none"> - Strength of Materials approach for Elastic analysis 	<ul style="list-style-type: none"> - Nonlinear Analysis - Analytical approach using differential equation solving - Numerical approach using Finite Element (FE) or Discrete Element (DE) solving 	<ul style="list-style-type: none"> - Beam or solid element - Rigid and Flexible Retaining Wall
Results	<ul style="list-style-type: none"> - Bending Moment - Shear Force - Support Reaction 	<ul style="list-style-type: none"> - Bending Moment - Shear Force - Support Reaction - Displacement of retaining wall 	<ul style="list-style-type: none"> - Bending Moment - Shear Force - Support Reaction - Displacement of retaining wall - Upstream Settlement

criteria are proposed, encompassing two dimensions of speed and effectiveness of the analyzed method, along with an additional dimension representing its efficiency. In contemporary contexts, the concepts of effectiveness and efficiency are widely used across various domains (including sociology, economics, medicine, and risk management). Yet, their apparent interchangeability can be misleading. The definitions of these terms are as follows:

Effectiveness signifies the capacity of a machine, an individual, or a method to achieve a goal regardless of the resources utilized. Conversely, efficiency pertains to the rational utilization of available resources to achieve predetermined objectives. In essence, it revolves around the ability to attain objectives while minimizing resource expenditure and time, ultimately leading to optimization.

The examination of the three calculation methods (LEM, SSI-SRM, and SSI-FEM) is grounded in a grading system that employs symbols ranging from (+ to +++). Evaluations based on six comparison criteria are detailed in **Table 9**.

Table 9. Comparison of sheet pile design methods based on efficiency criteria.

Criteria	LEM	SSI-SRM	SSI-FEM
Design dimension	+	+	+++
Modeling of soil, ISS, and soil-wall interface	+	++	+++
Modeling of the sheet pile structure	+	+++	+++
Modeling of tie rods/buttresses	+	++	++
Incorporation of phasing	+	+++	+++
Result	+	++	+++
TOTAL	6+	13+	17+

The cumulative score garnered from these criteria serves as a gauge to judge the effectiveness of each method. Consequently, this comparative analysis indicates that the SSI-FEM method is logically the most effective, followed by SSI-SRM, and then the LEM method.

When it comes to assessing the efficiency of these methods, an additional criterion pertaining to result satisfaction is introduced. This criterion establishes a threshold at which outcomes are deemed satisfactory. Coupled with this initial criterion is an estimation of computation time or resource allocation. Subsequent evaluation then hinges on minimizing these factors (by choosing the approach with the least computational time or resource usage).

For example, by setting a threshold for efficiency satisfaction at 10+ (as depicted in **Table 9**), both methods are considered effective, with a slightly more favorable evaluation for SSI-FEM. Nevertheless, due to SSI-FEM's higher computational time demands in comparison to SSI-SRM, it can be inferred that SSI-SRM demonstrates greater efficiency. This analytical process can be visually presented through **Figure 14**.

3.5. Mohr-Coulomb Constitutive Model

The Mohr-Coulomb law serves as the cornerstone for widely adopted soil design methodologies, underpinning geotechnical engineering and soil mechanics. Named after its creators, Otto Mohr and Charles-Augustin de Coulomb [12] [71] [72] [73], this model has gained extensive utility in understanding the intricate interactions between shear strength, stress-strain behaviors, and the responses of soils-rock masses.

At its core, the Mohr-Coulomb model characterizes material response under shear stress by establishing a linear connection between shear stress and normal stress along a designated plane within the material. This relationship is visually depicted through the Mohr-Coulomb failure envelope.

Two essential parameters, cohesion (c) and the angle of inner friction (ϕ), form the backbone of the Mohr-Coulomb model. Cohesion defines the shear stress axis interception of the failure envelope, denoting shear strength in the

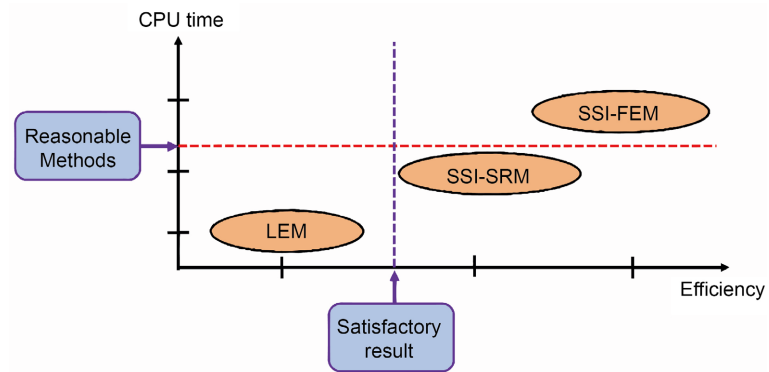


Figure 14. Comparative analysis of retaining wall design methods in terms of effectiveness and efficiency.

nonexistence of normal stress. The angle of inner friction represents the inclination of the failure envelope, encapsulating the material's resistance to shearing across the specified plane.

The Mohr-Coulomb failure criterion posits that failure hinges on maximum shear stress, correlated with normal stress. The mathematical representation of the failure envelope is expressed as:

$$\tau = c + \sigma \cdot \tan \phi \quad (26)$$

where:

- τ signifies shear stress on the designated plane;
- c represents the material's cohesion;
- σ indicates the normal stress applied on the plane;
- ϕ denotes the angle of inner friction.

The Mohr-Coulomb model finds wide application in geotechnical engineering, encompassing tasks such as slope stability evaluations, foundation design, and the planning of retaining walls and tunneling projects. While the model offers a simplified depiction of soil behavior, it plays a pivotal role as an initial tool for engineering analyses.

In practical terms, soil behavior is intrinsically non-linear and can be modeled using diverse methodologies within PLAXIS. In elastic contexts, soil adheres to Hooke's linear elastic law. When considering the stress situation at a specific point in the soil reaches failure thresholds, the material undergoes perfectly plastic yielding. This essentially means that the failure surface is wholly governed by the model's parameters and remains unaffected by plastic deformation [74].

It is imperative to recognize that the Mohr-Coulomb model has its limitations, particularly when addressing complex stress scenarios or materials exhibiting strain-softening behavior. In such cases, advanced constitutive models are necessary for an accurate portrayal of material behavior.

3.6. Mohr-Coulomb Constitutive Model: Limitations and Drawbacks

The calibration of the Mohr-Coulomb model involves determining the cohesion

parameter, c , and the internal friction angle, Φ . These parameters are obtained from several triaxial tests. Additionally, the dilation angle, Ψ , must be calibrated based on observations during the tests. The flow parameters are also obtained from a separate test.

The Mohr-Coulomb model has a substantial role to play in geotechnical engineering, but it's crucial to acknowledge its limitations and shortcomings, which engineers and researchers should take into account:

- **Assumption of Linear Behavior:** The Mohr-Coulomb model relies on the assumption of a linear correlation between shear stress and normal stress along a failure plane. However, this assumption doesn't universally apply, particularly for soil materials exhibiting non-linear or strain-softening characteristics. The inherent complexity and non-linearity of soil behavior can lead to potential inaccuracies when solely relying on the Mohr-Coulomb model for predictions.
- **Omission of Stress History Influence:** One of the notable drawbacks of the model is its neglect of stress history's impact on soil behavior. Soil response often bears the imprint of its past loading experiences, a factor overlooked by the Mohr-Coulomb model. This omission becomes particularly pertinent when dealing with soils subjected to varying stress conditions over time.
- **Inadequacy in Handling Anisotropy:** The Mohr-Coulomb model falls short in addressing anisotropic soils that manifest varying properties along different directions. Anchored in the assumption of isotropy, the model treats material properties as uniform across all directions. This simplification introduces significant discrepancies in scenarios where anisotropy plays a pivotal role.
- **Limited Suitability for Dynamic Analysis:** Primarily designed for static analysis, the Mohr-Coulomb model may not prove adequate in scenarios involving dynamic events like earthquakes or rapid loading conditions. Its scope doesn't encompass the time-dependent behavior of the time-related response of soils or their response to swift shifts in loading circumstances.
- **Challenges with Unsaturated Soils:** The model's scope is largely confined to saturated soils, potentially leading to inaccurate outcomes for unsaturated soils, which boast distinct mechanical properties due to varying water content levels. To aptly mirror the behavior of unsaturated soils, the adoption of more advanced models is indispensable.
- **Absence of Strain Localization Consideration:** The Mohr-Coulomb model is rooted in the assumption of uniform stress and strain distribution across a failure plane. However, this simplification sidesteps the potential occurrence of strain localization, where deformation concentrates in localized zones. The model struggles to accurately capture this intricate phenomenon.
- **Exclusion of Progressive Failure Dynamics:** The model disregards progressive failure mechanisms that some soil conditions exhibit. It relies on the concept that failure is instantaneous upon surpassing a shear stress threshold, a pers-

pective that may not accurately portray the gradual failure process observed in real-world scenarios.

- **Applicability to Non-Cohesive Soils:** The Mohr-Coulomb model aligns better with cohesive soils but may fall short in effectively representing the behavior of non-cohesive substances like sands and gravels. The model's parameters might not adequately encapsulate the unique stress-strain relationships inherent to such materials.

In summation, while the Mohr-Coulomb model remains a cornerstone of geotechnical engineering, it's crucial to recognize and grapple with its limitations. Engineers need to exercise prudence by considering soil characteristics and loading conditions judiciously. Exploring more advanced constitutive models becomes indispensable when tackling intricate or non-linear soil behaviors.

4. Conclusions

In this extensive examination of “Innovative Techniques Unveiled in Advanced Sheet Pile Curtain Design”, the study delved into the complexities of geotechnical engineering, focusing on soil-structure interaction (SSI). The investigation centered on sheet pile design, highlighting two primary methodological categories: Limit Equilibrium Methods (LEM) and Soil-Structure Interaction Methods (SSIM).

LEM methods, rooted in classical principles and analytical simplicity, serve as valuable tools for preliminary design considerations. However, their limitations become apparent in addressing the intricate complexities of real-world soil-structure interaction. Conversely, SSIM methods, exemplified by the SSI-SR approach, offer precision and depth. Leveraging numerical techniques such as Finite Element (FE) and Finite Difference (FD) analyses, these methods empower engineers to navigate the multifaceted dynamics of soil-structure interaction.

The exploration extended into the realm of SSI-FE, uncovering its significant role in civil engineering. By integrating Finite Element analysis with considerations for soil-structure interaction, the SSI-FE method provides engineers with a holistic understanding of structural interaction with the dynamic geotechnical environment.

In the pursuit of precision, the article systematically examined critical components governing SSIM methods, including reaction laws (RL), P - Y relationships, and elastic stiffness (K_{ss}). These insights furnish engineers with the necessary tools to navigate the complex geotechnical design landscape.

Significantly, the study acknowledged the importance of the Mohr-Coulomb constitutive model while candidly recognizing its limitations. This balanced perspective guides practitioners in making informed decisions during geotechnical analyses, emphasizing the consideration of advanced models in complex scenarios.

As this paper concludes its exploration, the future of geotechnical engineering is recognized as one of continuous learning and innovation. Armed with ad-

vancing technology and a deepening understanding of soil-structure interaction, the scientific community moves forward, prepared to address the evolving challenges of the engineering landscape. The commitment remains focused on ensuring the safety, stability, and efficiency of geotechnical structures through cutting-edge design and analysis techniques.

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Data Availability Statement

Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

Conflict of Interest Statement

On behalf of all authors, the corresponding author states that there is no competing interests regarding the publication of this research.

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