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An Extended Pranav Distribution

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Authors' contributions

This work was carried out in collaboration between all authors. Author ORU designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. Authors ORU, NPA and UCO managed the analyses of the study. Author ORU managed the literature searches. All authors read and approved the final manuscript.

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Abstract

In this study, we proposed a generalization of the Pranav distribution by Shukla (2018). This new distribution called an extended Pranav distribution is obtained using the exponentiation method. The statistical characteristics of this new distribution such as the moments, moment generating function, reliability function, hazard function, Rényi entropy and order statistics are derived. The graphical illustrations of the shapes of the probability density function, the cumulative distribution function, and hazard rate functions are provided. The maximum likelihood estimates of the parameters were obtained and finally, we examine the performance of this new distribution using some real-life data sets to show its flexibility and better goodness of fit as compared with other distributions.

Keywords: Pranav distribution; exponentiation; hazard rate function; Rényi entropy.

1 Introduction

In order to study lifetime data in real life, two things often stand out- the analysis and modeling of the datasets using a probability distribution. This is usually done with the aim of finding the true nature of the data set and how it behaves. Hence, over the years, researchers have proposed a lot of distributions to

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achieve this aim over the various field of study such as applied sciences, engineering, finance, insurance, etc. because of the peculiarity of the data sets. These data sets often possess different shapes, mean residual life, hazard rates etc. and requires a unique distribution which gives the best fit.

A lot of lifetime distributions have been proposed over the years for analyzing and modeling lifetime datasets. Distributions such as the exponential and Lindley distributions have been proposed and applied to some lifetime distributions. Shanker [1] carried out a comparative study on the exponential and Lindley distributions using lifetime data sets obtained in the field of engineering and biomedical science. Their findings showed that the two distributions were not suitable for most of the lifetime datasets.

Hence the quest to seek out the best lifetime distribution has led many researchers to propose a lot of new lifetime distributions for a better fit to lifetime datasets. Shanker [1] proposed the one parameter Akash distribution for modeling lifetime datasets. The author stated that the Akash distribution was superior to lifetime distributions such as the Weibull, Exponential, and Lindley distributions. Other lifetime distributions developed over the years include the Power Lindley distribution [2], the inverse Weibull distribution [3], Sujatha distribution [4], Two-parameter Akash distribution [5], Discrete Shanker distribution [6] among others. Recently, Shukla [7] proposed a new lifetime distribution known as Pranav distribution for modeling lifetime datasets. The author demonstrated the ability of this distribution to provide a better fit to some real-life data sets than some of the distributions mentioned earlier such as the Akash, Lindley, and Sujatha among others.

The lifetime distributions above are referred to as baseline distributions because, in order to ensure their robustness, they are often generalized. The exponentiation method of Mudholkar and Srivastava [8] is popularly used when generalizing a baseline distribution due to its ability to be more flexible and give a better fit. Some of the distributions that have been generalized using this method by researchers include the exponentiated Weibull distribution [9], the exponentiated Lindley distribution [10], the exponentiated Fréchet distribution [11], exponentiated Exponential distribution [12], the exponentiated power Lindley distribution [13], exponentiated Inverted Weibull distribution [14] among others. These exponentiated distributions were shown to be superior and more flexible than their baseline distribution using real-life data sets.

The aim of this study, therefore, is to improve on the Pranav distribution using the exponentiation technique. In order to achieve this, we propose a new distribution with its mathematical properties called an extended Pranav distribution, a generalization of the Pranav distribution. Furthermore, we will show the flexibility of this new distribution as well as its ability to provide a better fit for some real-life data sets.

2 The Exponentiated Pranav Distribution

Recently, Shukla [7] proposed a one parameter lifetime distribution for modelling lifetime data sets. The pdf and cdf of the Pranav distribution as proposed by Shukla (2018) is given respectively as

$$f(x) = \frac{\theta^4}{\theta^4+6}(\theta + x^3)e^{-\theta x};\ x > 0, \theta > 0, \tag{1}$$

$$F(x) = 1 - \left[1 + \frac{\theta x(\theta^2 x^2 + 3\theta x + 6)}{\theta^4+6}\right]e^{-\theta x};\ x > 0, \theta > 0. \tag{2}$$

A random variable X is said to have an exponentiated distribution if the cdf and pdf are given respectively by

$$Q(x) = [F(x)]^{\alpha}; X \in R^{'}, \alpha > 0 \tag{3}$$

$$q(x) = \alpha[F(x)]^{\alpha-1}f(x) \tag{4}$$

Hence, we obtain the pdf and cdf of the new extended Pranav distribution using the equations above as

$$Q(x) = \left[1 - \left[1 + \frac{\theta x(\theta^2 x^2 + 3\theta x + 6)}{\theta^4 + 6}\right] e^{-\theta x}\right]^{\alpha}; x > 0, \theta > 0, \alpha > 0. \tag{5}$$

$$q(x) = \alpha \frac{\theta^4}{\theta^4 + 6}(\theta + x^3)\left[1 - \left(1 + \frac{\theta x(\theta^2 x^2 + 3\theta x + 6)}{\theta^4 + 6}\right) e^{-\theta x}\right]^{\alpha - 1} e^{-\theta x}; x > 0, \theta > 0, \alpha > 0, \tag{6}$$

where θ and α are the scale and shape parameters respectively.

The plots of the probability density function and cumulative distribution function are shown in Figs. 1 and 2 for different values of the parameters.

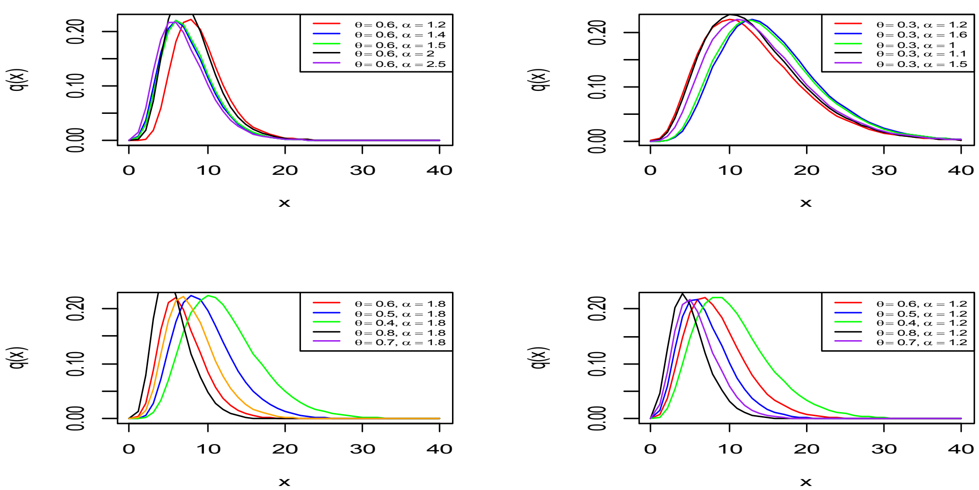

Fig. 1. The PDF of the extended Pranav distribution

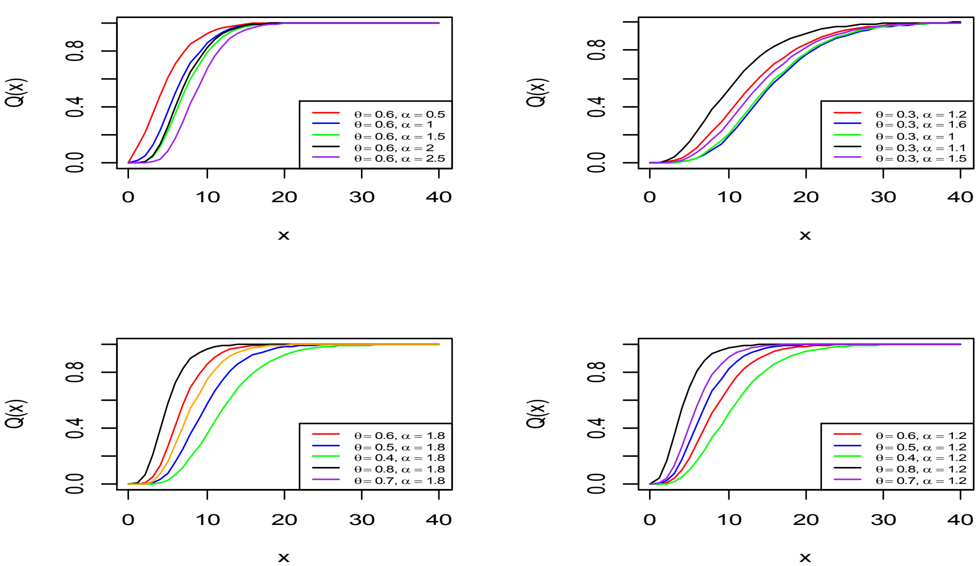

Fig. 2. The CDF of the extended Pranav distribution

3 Mathematical Characteristics

In this section, we present some of the mathematical properties of the extended Pranav distribution such as the moment and moment generating function, the order statistics, entropy and reliability analysis.

3.1 Moments

We derive the rth moment of the extended Pranav distribution in this subsection.

Theorem I

Given a random variable X, following an extended Pranav distribution, the rth order moment about origin, $E(X^r)$ of the extended Pranav distribution is given by

$$E(X^r) = A_{i,j,k,l}\frac{(r+3j-k-l)!}{(i+1)^{r+3j-k-l+1}} + B_{i,j,k,l}\frac{(r+3j-k-l+3)!}{(i+1)^{r+3j-k-l+4}}, \quad (7)$$

Where

$A_{i,j,k,l} = \sum_{i=0}^{\infty}\binom{\alpha-1}{i}(-1)^i\sum_{j=0}^{\infty}\binom{i}{j}\sum_{k=0}^{j}\binom{j}{k}\sum_{l=0}^{k}\binom{k}{l}\frac{3^k\cdot 2^l\cdot\alpha\cdot\theta^{-r+4}}{(\theta^4+6)^{j+1}}$ and $B_{i,j,k,l} = \sum_{i=0}^{\infty}\binom{\alpha-1}{i}(-1)^i\sum_{j=0}^{\infty}\binom{i}{j}\sum_{k=0}^{j}\binom{j}{k}\sum_{l=0}^{k}\binom{k}{l}\frac{3^k\cdot 2^l\cdot\alpha\cdot\theta^{-r}}{(\theta^4+6)^{j+1}}$.

Proof

The rth moment of a random variable X is given by;

$$E(X^r) = \int_0^\infty x^r q(x)dx$$

$$= \int_0^\infty x^r\frac{\alpha\theta^4}{\theta^4+6}(\theta+x^3)\left[1-\left(1+\frac{\theta x(\theta^2x^2+3\theta x+6)}{\theta^4+6}\right)e^{-\theta x}\right]^{\alpha-1}e^{-\theta x}dx$$

$$= \int_0^\infty \frac{\alpha\theta^5x^r}{\theta^4+6}e^{-\theta x}\left[1-\left(1+\frac{\theta x(\theta^2x^2+3\theta x+6)}{\theta^4+6}\right)e^{-\theta x}\right]^{\alpha-1}dx$$

$$+\int_0^\infty \frac{\alpha\theta^4x^{r+3}}{\theta^4+6}e^{-\theta x}\left[1-\left(1+\frac{\theta x(\theta^2x^2+3\theta x+6)}{\theta^4+6}\right)e^{-\theta x}\right]^{\alpha-1}dx$$

Using the Binomial series expansion below,

$$\left[1-\left(1+\frac{\theta x(\theta^2x^2+3\theta x+6)}{\theta^4+6}\right)e^{-\theta x}\right]^{\alpha-1} = \sum_{i=0}^{\infty}\binom{\alpha-1}{i}(-1)^i\left(1+\frac{\theta x(\theta^2x^2+3\theta x+6)}{\theta^4+6}\right)^i e^{-i\theta x},$$

and the binomial expansion of $\left[1+\frac{\theta x(\theta x+2)}{\theta^2+2}\right]^i$ given by

$$\left[1+\frac{\theta x(\theta^2x^2+3\theta x+6)}{\theta^4+6}\right]^i = \sum_{j=0}^{\infty}\binom{i}{j}\left[\frac{\theta x(\theta^2x^2+3\theta x+6)}{\theta^4+6}\right]^j,$$

$$= \sum_{j=0}^{\infty}\binom{i}{j}\frac{\theta^jx^j}{(\theta^4+6)^j}\sum_{k=0}^{j}\binom{j}{k}\sum_{l=0}^{k}\binom{k}{l}3^k\cdot 2^l\cdot\theta^{2j-k-l}\cdot x^{2j-k-l}$$

$$\therefore \left[1-\left(1+\frac{\theta x(\theta^2x^2+3\theta x+6)}{\theta^4+6}\right)e^{-\theta x}\right]^{\alpha-1} = \sum_{i=0}^{\infty}\binom{\alpha-1}{i}(-1)^i\sum_{j=0}^{\infty}\binom{i}{j}\sum_{k=0}^{j}\binom{j}{k}\sum_{l=0}^{k}\binom{k}{l}\cdot\frac{\theta^jx^j}{(\theta^4+6)^j}\cdot 3^k\cdot 2^l\cdot\theta^{2j-k-l}\cdot x^{2j-k-l}.$$

Substituting, we have

$$E(X^r) = \sum_{i=0}^{\infty}\binom{\alpha-1}{i}(-1)^i\sum_{j=0}^{\infty}\binom{i}{j}\sum_{k=0}^{j}\binom{j}{k}\sum_{l=0}^{k}\binom{l}{k}\frac{\alpha\cdot 3^k\cdot 2^l\cdot\theta^{3j-k-l+5}}{(\theta^4+6)^{j+1}}\int_0^{\infty}x^{r+3j-k-l}e^{-\theta x(i+1)}dx + \sum_{i=0}^{\infty}\binom{\alpha-1}{i}(-1)^i\sum_{j=0}^{\infty}\binom{i}{j}\sum_{k=0}^{j}\binom{j}{k}\sum_{l=0}^{k}\binom{l}{k}\frac{\alpha\cdot 3^k\cdot 2^l\cdot\theta^{3j-k-l+4}}{(\theta^4+6)^{j+1}}\int_0^{\infty}x^{r+3j-k-l+3}e^{-\theta x(i+1)}dx$$

Recall $\int_0^{\infty}x^n e^{-ax}dx = \frac{\Gamma(n+1)}{a^{n+1}}$ and $\Gamma(\alpha) = (\alpha-1)!$

$E(X^r) = \sum_{i=0}^{\infty}\binom{\alpha-1}{i}(-1)^i\sum_{j=0}^{\infty}\binom{i}{j}\sum_{k=0}^{j}\binom{j}{k}\sum_{l=0}^{k}\binom{l}{k}\frac{\alpha\cdot 3^k\cdot 2^l\cdot\theta^{3j-k-l+5}}{(\theta^4+6)^{j+1}}\cdot\theta^{-(r+3j-k-l+1)}\frac{(r+3j-k-l)!}{[i+1]^{r+3j-k-l+1}} + \sum_{i=0}^{\infty}\binom{\alpha-1}{i}(-1)^i\sum_{j=0}^{\infty}\binom{i}{j}\sum_{k=0}^{j}\binom{j}{k}\sum_{l=0}^{k}\binom{l}{k}\frac{\alpha\cdot 3^k\cdot 2^l\cdot\theta^{3j-k-l+4}}{(\theta^4+6)^{j+1}}\cdot\theta^{-(r+3j-k-l+4)}\frac{(r+3j-k-l+3)!}{[i+1]^{r+3j-k-l+4}}.$

Let

$$A_{i,j,k,l} = \sum_{i=0}^{\infty}\binom{\alpha-1}{i}(-1)^i\sum_{j=0}^{\infty}\binom{i}{j}\sum_{k=0}^{j}\binom{j}{k}\sum_{l=0}^{k}\binom{k}{l}\frac{3^k\cdot 2^l\cdot\alpha\cdot\theta^{-r+4}}{(\theta^4+6)^{j+1}}\ and\ B_{i,j,k,l} = \sum_{i=0}^{\infty}\binom{\alpha-1}{i}(-1)^i\sum_{j=0}^{\infty}\binom{i}{j}\sum_{k=0}^{j}\binom{j}{k}\sum_{l=0}^{k}\binom{k}{l}\frac{3^k\cdot 2^l\cdot\alpha\cdot\theta^{-r}}{(\theta^4+6)^{j+1}}$$

Therefore,

$E(X^r) = A_{i,j,k,l}\frac{(r+3j-k-l)!}{(i+1)^{r+3j-k-l+1}} + B_{i,j,k,l}\frac{(r+3j-k-l+3)!}{(i+1)^{r+3j-k-l+4}}.$

3.2 Moment generating function

Here, we propose the moment generating function for the extended Pranav distribution.

Theorem II

Let X have an extended Pranav distribution. Then the moment generating function of X,$M_X(t)$ is given by

$$M_X(t) = \sum_{k=0}^{\infty}\left(\frac{t}{\theta}\right)^k\left\{A_{i,j,l,m}\frac{(k+3j-l-m)!}{k![i+1]^{k+3j-l-m+1}} + B_{i,j,l,m}\frac{(k+3j-l-m+3)!}{k![i+1]^{k+3j-l-m+4}}\right\}, \quad (8)$$

where

$A_{i,j,l,m} = \sum_{i=0}^{\infty}\binom{\alpha-1}{i}(-1)^i\sum_{j=0}^{\infty}\binom{i}{j}\sum_{l=0}^{j}\binom{j}{l}\sum_{m=0}^{l}\binom{l}{m}\frac{3^l\cdot 2^m\cdot\alpha}{(\theta^4+6)^{j+1}}\ and\ B_{i,j,l,m} = \sum_{i=0}^{\infty}\binom{\alpha-1}{i}(-1)^i\sum_{j=0}^{\infty}\binom{i}{j}\sum_{l=0}^{j}\binom{j}{l}\sum_{m=0}^{l}\binom{l}{m}\frac{3^l\cdot 2^m\cdot\alpha}{\theta^4(\theta^4+6)^{j+1}}.$

Proof

The moment generating function of a random variable X is given by

$$M_x(t) = E(e^{tx}) = \int_0^\infty e^{tx} q(x)dx$$

$$= \int_0^\infty e^{tx} \frac{\alpha\theta^4}{\theta^4+6}(\theta + x^3)\left[1 - \left(1 + \frac{\theta x(\theta^2 x^2 + 3\theta x + 6)}{\theta^4+6}\right)e^{-\theta x}\right]^{\alpha-1} e^{-\theta x}dx$$

$$= \int_0^\infty \frac{\alpha\theta^5}{\theta^4+6} e^{-x(\theta-t)}\left[1 - \left(1 + \frac{\theta x(\theta^2 x^2 + 3\theta x + 6)}{\theta^4+6}\right)e^{-\theta x}\right]^{\alpha-1} dx$$

$$+ \int_0^\infty \frac{\alpha\theta^4 x^3}{\theta^4+6} e^{-x(\theta-t)}\left[1 - \left(1 + \frac{\theta x(\theta^2 x^2 + 3\theta x + 6)}{\theta^4+6}\right)e^{-\theta x}\right]^{\alpha-1} dx$$

Using the Binomial series expansion below,

$$\left[1 - \left(1 + \frac{\theta x(\theta^2 x^2+3\theta x+6)}{\theta^4+6}\right)e^{-\theta x}\right]^{\alpha-1} = \sum_{i=0}^{\infty}\binom{\alpha-1}{i}(-1)^i\left(1 + \frac{\theta x(\theta^2 x^2+3\theta x+6)}{\theta^4+6}\right)^i e^{-i\theta x},$$

and the binomial expansion of $\left[1 + \frac{\theta x(\theta x+2)}{\theta^2+2}\right]^i$ given by

$$\left[1 + \frac{\theta x(\theta^2 x^2+3\theta x+6)}{\theta^4+6}\right]^i = \sum_{j=0}^{\infty}\binom{i}{j}\left[\frac{\theta x(\theta^2 x^2+3\theta x+6)}{\theta^4+6}\right]^j,$$

$$= \sum_{j=0}^{\infty}\binom{i}{j}\sum_{l=0}^{j}\binom{j}{l}\sum_{m=0}^{l}\binom{l}{m}\frac{\theta^j x^j}{(\theta^4+6)^j}\cdot 3^l \cdot 2^m \cdot \theta^{2j-l-m}\cdot x^{2j-l-m}$$

$$\therefore \left[1 - \left(1 + \frac{\theta x(\theta^2 x^2+3\theta x+6)}{\theta^4+6}\right)e^{-\theta x}\right]^{\alpha-1} = \sum_{i=0}^{\infty}\binom{\alpha-1}{i}(-1)^i\sum_{j=0}^{\infty}\binom{i}{j}\sum_{l=0}^{j}\binom{j}{l}\sum_{m=0}^{l}\binom{l}{m}\cdot\frac{\theta^j x^j}{(\theta^4+6)^j}\cdot 3^l\cdot 2^m\cdot \theta^{2j-l-m}\cdot x^{2j-l-m}.$$

$Also, e^{tx} = \sum_{k=0}^{\infty}\frac{(tx)^k}{k!}$. Substituting, we have

$$M_X(t) = \sum_{i=0}^{\infty}\binom{\alpha-1}{i}(-1)^i\sum_{j=0}^{\infty}\binom{i}{j}\sum_{l=0}^{j}\binom{j}{l}\sum_{m=0}^{l}\binom{l}{m}\sum_{k=0}^{\infty}\frac{t^k}{k!}\frac{3^l\cdot 2^m\cdot\alpha\cdot\theta^{3j-l-m+5}}{(\theta^4+6)^{j+1}}\int_0^\infty x^{k+3j-l-m}e^{-\theta x(i+1)}dx$$
$$+\sum_{i=0}^{\infty}\binom{\alpha-1}{i}(-1)^i\sum_{j=0}^{\infty}\binom{i}{j}\sum_{l=0}^{j}\binom{j}{l}\sum_{m=0}^{l}\binom{l}{m}\sum_{k=0}^{\infty}\frac{t^k}{k!}\frac{3^l\cdot 2^m\cdot\alpha\cdot\theta^{3j-l-m+4}}{(\theta^4+6)^{j+1}}\int_0^\infty x^{k+3j-l-m+3}e^{-\theta x(i+1)}dx$$

Recall $\int_0^\infty x^n e^{-ax}dx = \frac{\Gamma(n+1)}{a^{n+1}}$ and $\Gamma(\alpha) = (\alpha-1)!$ Hence

$$M_x(t) = \sum_{i=0}^{\infty}\binom{\alpha-1}{i}(-1)^i\sum_{j=0}^{\infty}\binom{i}{j}\sum_{l=0}^{j}\binom{j}{l}\sum_{m=0}^{l}\binom{l}{m}\sum_{k=0}^{\infty}\frac{t^k}{k!}\frac{3^l\cdot 2^m\cdot\alpha\cdot\theta^{3j-l-m+5}}{(\theta^4+6)^{j+1}}\cdot\theta^{-(k+3j-l-m+1)}\frac{(k+3j-l-m)!}{[i+1]^{k+3j-l-m+1}} +$$
$$\sum_{i=0}^{\infty}\binom{\alpha-1}{i}(-1)^i\sum_{j=0}^{\infty}\binom{i}{j}\sum_{l=0}^{j}\binom{j}{l}\sum_{m=0}^{l}\binom{l}{m}\sum_{k=0}^{\infty}\frac{t^k}{k!}\frac{3^l\cdot 2^m\cdot\alpha\cdot\theta^{3j-l-m+4}}{(\theta^4+6)^{j+1}}\cdot\theta^{-(k+3j-l-m+4)}\frac{(k+3j-l-m+3)!}{[i+1]^{k+3j-l-m+4}}.$$

$M_x(t) =$
$\sum_{i=0}^{\infty}\binom{\alpha-1}{i}(-1)^i \sum_{j=0}^{\infty}\binom{i}{j}\sum_{l=0}^{j}\binom{j}{l}\sum_{m=0}^{l}\binom{l}{m}\sum_{k=0}^{\infty}\left(\frac{t}{\theta}\right)^k \frac{3^l \cdot 2^m \cdot \alpha \cdot \theta^{-4}}{k!(\theta^4+6)^{j+1}} \cdot \frac{(k+3j-l-m)!}{[i+1]^{k+3j-l-m+1}} +$
$\sum_{i=0}^{\infty}\binom{\alpha-1}{i}(-1)^i \sum_{j=0}^{\infty}\binom{i}{j}\sum_{l=0}^{j}\binom{j}{l}\sum_{m=0}^{l}\binom{l}{m}\sum_{k=0}^{\infty}\left(\frac{t}{\theta}\right)^k \frac{3^l \cdot 2^m \cdot \alpha}{(\theta^4+6)^{j+1}} \cdot \frac{(k+3j-l-m+3)!}{k![i+1]^{k+3j-l-m+4}}.$

Let

$$A_{i,j,l,m} = \sum_{i=0}^{\infty}\binom{\alpha-1}{i}(-1)^i \sum_{j=0}^{\infty}\binom{i}{j}\sum_{l=0}^{j}\binom{j}{l}\sum_{m=0}^{l}\binom{l}{m}\frac{3^l \cdot 2^m \cdot \alpha}{(\theta^4+6)^{j+1}} \; and \; B_{i,j,l,m} = \sum_{i=0}^{\infty}\binom{\alpha-1}{i}(-1)^i \sum_{j=0}^{\infty}\binom{i}{j}\sum_{l=0}^{j}\binom{j}{l}\sum_{m=0}^{l}\binom{l}{m}\frac{3^l \cdot 2^m \cdot \alpha}{\theta^4(\theta^4+6)^{j+1}}$$

Therefore,

$M_X(t) = \sum_{k=0}^{\infty}\left(\frac{t}{\theta}\right)^k \left\{A_{i,j,l,m}\frac{(k+3j-l-m)!}{k![i+1]^{k+3j-l-m+1}} + B_{i,j,l,m}\frac{(k+3j-l-m+3)!}{k![i+1]^{k+3j-l-m+4}}\right\}$

3.3 Order statistics

The order statistics of the extended Pranav distribution is presented below.

Theorem III

Suppose $X_1, X_2, \dots, X_n$ is a random sample from an extended Pranav distribution. Let $X_{(1)}, X_{(2)}, \dots, X_{(n)}$ denote the corresponding order statistics. Then, the probability density function, pdf of the pth order statistics, say $X = X_{(p)}$, is given by

$f_X(x) =$
$\frac{\alpha n!(\theta+x^3)e^{-\theta x(j+1)}}{(p-1)!(n-p)!}\sum_{i=0}^{n-p}\binom{n-p}{i}(-1)^i \sum_{j=0}^{\infty}\binom{\alpha(p+i)}{j}(-1)^j \sum_{k=0}^{\infty}\binom{j}{k}\sum_{l=0}^{k}\binom{k}{l}\sum_{m=0}^{l}\binom{l}{m} \times \frac{3^l \cdot 2^m \cdot \theta^{3k-l-m+4}x^{3k-l-m}}{(\theta^4+6)^{k+1}}$ (9)

While the cdf is given by;

$F_X(x) =$
$\sum_{i=p}^{n}\binom{n}{i}\sum_{j=0}^{n-i}\binom{n-i}{j}(-1)^j \sum_{k=0}^{\infty}\binom{\alpha(j+1)-1}{k}(-1)^k \sum_{l=0}^{\infty}\binom{k}{l}\sum_{m=0}^{l}\binom{l}{m}\sum_{n=0}^{m}\binom{m}{n} \times \frac{3^m \cdot 2^n \cdot \theta^{3l-m-n}x^{3l-m-n}e^{-i\theta x}}{(\theta^4+6)^{k+1}}$ (10)

Proof.

The pdf of a pth order statistics is given by

$$f_X(x) = \frac{n!}{(p-1)!\,(n-p)!}Q^{p-1}(x)[1-Q(x)]^{n-p}q(x)$$

$f_X(x) = \frac{n!}{(p-1)!(n-p)!}\sum_{i=0}^{n-p}\binom{n-p}{i}(-1)^i Q^{p-1+i}(x)q(x)$ (11)

Substituting for $Q(x)$ and $q(x)$ in the equations above we obtain the pdf and cdf of the order statistics respectively as;

$$f_X(x) = \frac{n!}{(p-1)!\,(n-p)!}\sum_{i=0}^{n-p}\binom{n-p}{i}(-1)^i\left[1-\left(1+\frac{\theta x(\theta^2x^2+3\theta x+6)}{\theta^4+6}\right)e^{-\theta x}\right]^{\alpha(p+i)-1} \cdot \frac{\alpha\theta^4}{\theta^4+6}(\theta+x^3)\left[1-\left(1+\frac{\theta x(\theta^2x^2+3\theta x+6)}{\theta^4+6}\right)e^{-\theta x}\right]^{\alpha-1} e^{-\theta x}$$

$$f_Y(y) = \frac{\alpha\theta^4 n!(\theta+x^3)e^{-\theta x}}{\theta^4+6\cdot(p-1)!(n-p)!}\sum_{i=0}^{n-p}\binom{n-p}{i}(-1)^i\left[1-\left(1+\frac{\theta x(\theta^2x^2+3\theta x+6)}{\theta^4+6}\right)e^{-\theta x}\right]^{\alpha(p+i)-1}.$$

Using the Binomial series expansion below,

$$\left[1-\left(1+\frac{\theta x(\theta^2x^2+3\theta x+6)}{\theta^4+6}\right)e^{-\theta x}\right]^{\alpha(p+j)-1} = \sum_{j=0}^{\infty}\binom{\alpha(p+j)-1}{j}(-1)^j\left(1+\frac{\theta x(\theta^2x^2+3\theta x+6)}{\theta^4+6}\right)^j e^{-j\theta x},$$

and the binomial expansion of $\left[1+\frac{\theta x(\theta x+2)}{\theta^2+2}\right]^j$ given by

$$\left[1+\frac{\theta x(\theta^2x^2+3\theta x+6)}{\theta^4+6}\right]^j = \sum_{k=0}^{\infty}\binom{j}{k}\left[\frac{\theta x(\theta^2x^2+3\theta x+6)}{\theta^4+6}\right]^k,$$

$$= \sum_{k=0}^{\infty}\binom{j}{k}\sum_{l=0}^{k}\binom{k}{l}\sum_{m=0}^{l}\binom{l}{m}\frac{\theta^k x^k}{(\theta^4+6)^k}\cdot 3^l\cdot 2^m\cdot\theta^{2k-l-m}\cdot x^{2k-l-m}$$

$$\therefore \left[1-\left(1+\frac{\theta x(\theta^2x^2+3\theta x+6)}{\theta^4+6}\right)e^{-\theta x}\right]^{\alpha(p+j)-1} = \sum_{j=0}^{\infty}\binom{\alpha(p+j)-1}{j}(-1)^j\sum_{k=0}^{\infty}\binom{j}{k}\sum_{l=0}^{k}\binom{k}{l}\sum_{m=0}^{l}\binom{l}{m}\cdot\frac{\theta^k x^k}{(\theta^4+6)^k}\cdot 3^l\cdot 2^m\cdot\theta^{2k-l-m}\cdot x^{2k-l-m}e^{-j\theta x}.$$

Therefore,

$$f_X(x) = \frac{\alpha n!\,(\theta+x^3)e^{-\theta x(j+1)}}{(p-1)!\,(n-p)!}\sum_{i=0}^{n-p}\binom{n-p}{i}(-1)^i\sum_{j=0}^{\infty}\binom{\alpha(p+i)}{j}(-1)^j\sum_{k=0}^{\infty}\binom{j}{k}\sum_{l=0}^{k}\binom{k}{l}\sum_{m=0}^{l}\binom{l}{m}\frac{3^l\cdot 2^m\cdot\theta^{3k-l-m+4}x^{3k-l-m}}{(\theta^4+6)^{k+1}}$$

The cdf of the p[th] order statistics is given by

$$F_X(x) = \sum_{i=p}^{n}\binom{n}{i}Q^i(x)\,[1-Q(x)]^{n-i}$$

$$F_X(x) = \sum_{i=p}^{n}\sum_{j=0}^{n-i}\binom{n}{i}\binom{n-i}{j}(-1)^j\,Q^{i+1}(x) \qquad (12)$$

$$F_X(x) = \sum_{i=p}^{n}\sum_{j=0}^{n-i}\binom{n}{i}\binom{n-i}{j}(-1)^j\left[1-\left[1+\frac{\theta x(\theta^2x^2+3\theta x+6)}{\theta^4+6}\right]e^{-\theta x}\right]^{\alpha(i+1)}$$

$$F_X(x) = \sum_{i=p}^{n}\binom{n}{i}\sum_{j=0}^{n-i}\binom{n-i}{j}(-1)^j\sum_{k=0}^{\infty}\binom{\alpha(j+1)-1}{k}(-1)^k\sum_{l=0}^{\infty}\binom{k}{l}\sum_{m=0}^{l}\binom{l}{m}\sum_{n=0}^{m}\binom{m}{n}\frac{3^m\cdot 2^n\cdot\theta^{3l-m-n}x^{3l-m-n}e^{-k\theta x}}{(\theta^4+6)^{k+1}}.$$

3.4 Entropy

Entropy measures the uncertainties associated with a random variable of a probability distributions. One of the type of entropy widely used is the Rényi's entropy [15].

Theorem IV

Given a random variable X, which follows an extended Pranav distribution. The Rényi entropy is given by

$$T_R(\beta) = \frac{1}{1-\beta} log \left\{ A_{i,j,k,l} \frac{(3j-k-l)!}{[i+1]^{3j-k-l+1}} + B_{i,j,k,l} \frac{(3\beta+3j-k-l)!}{[i+1]^{3\beta+3j-k-l+1}} \right\} \tag{13}$$

where

$A_{i,j,k,l} = \sum_{i=0}^{\infty} \binom{\beta(\alpha-1)-1}{i} (-1)^i \sum_{j=0}^{\infty} \binom{i}{j} \sum_{k=0}^{j} \binom{j}{k} \sum_{l=0}^{k} \binom{k}{l} \frac{3^k \cdot 2^l \cdot \alpha^\beta \cdot \theta^{5\beta-1}}{(\theta^4+6)^{\beta+j}}$ *and* $B_{i,j,k,l} =$
$\sum_{i=0}^{\infty} \binom{\beta(\alpha-1)-1}{i} (-1)^i \sum_{j=0}^{\infty} \binom{i}{j} \sum_{k=0}^{j} \binom{j}{k} \sum_{l=0}^{k} \binom{k}{l} \frac{3^k \cdot 2^l \cdot \alpha^\beta \cdot \theta^{\beta-1}}{(\theta^4+6)^{\beta+j}}$.

Proof.

The Rényi entropy is given by;

$$T_R(\beta) = \frac{1}{1-\beta} log \left[\int q^\beta(x) dx \right];\ \beta > 0 \ and\ \beta \neq 1$$

$$T_R(\beta) = \frac{1}{1-\beta} log \left\{ \int_0^\infty \left[\frac{\alpha\theta^4}{\theta^4+6} (\theta + x^3) \left[1 - \left(1 + \frac{\theta x(\theta^2 x^2 + 3\theta x + 6)}{\theta^4 + 6} \right) e^{-\theta x} \right]^{\alpha-1} e^{-\theta x} \right]^\beta dx \right\}$$

$$T_R(\beta) = \frac{1}{1-\beta} log \left\{ \int_0^\infty \frac{\alpha^\beta \theta^{4\beta}}{(\theta^4+6)^\beta} (\theta + x^3)^\beta \left[1 - \left(1 + \frac{\theta x(\theta^2 x^2 + 3\theta x + 6)}{\theta^4 + 6} \right) e^{-\theta x} \right]^{\beta(\alpha-1)} e^{-\theta\beta x} dx \right\}$$

$$T_R(\beta) = \frac{1}{1-\beta} log \left\{ \int_0^\infty \frac{\alpha^\beta \theta^{5\beta}}{(\theta^4+6)^\beta} \left[1 - \left(1 + \frac{\theta y(\theta^2 x^2 + 3\theta x + 6)}{\theta^4 + 6} \right) e^{-\theta x} \right]^{\beta(\alpha-1)} e^{-\theta\beta x} dx + \int_0^\infty \frac{\alpha^\beta \theta^{4\beta} x^{3\beta}}{(\theta^4+6)^\beta} \left[1 - \left(1 + \frac{\theta x(\theta^2 x + 3\theta x + 6)}{\theta^4 + 6} \right) e^{-\theta x} \right]^{\beta(\alpha-1)} e^{-\theta\beta x} dx \right\}$$

Using the following Binomial series expansion,

$\left[1 - \left(1 + \frac{\theta x(\theta^2 x^2 + 3\theta x + 6)}{\theta^4+6} \right) e^{-\theta x} \right]^{\beta(\alpha-1)} = \sum_{i=0}^{\infty} \binom{\beta(\alpha-1)-1}{i} (-1)^i \left(1 + \frac{\theta x(\theta^2 x^2 + 3\theta x + 6)}{\theta^4+6} \right)^i e^{-i\theta x}$,

and the binomial expansion of $\left[1 + \frac{\theta x(\theta x+2)}{\theta^2+2} \right]^i$ given by

$$\left[1 + \frac{\theta x(\theta^2 x^2 + 3\theta x + 6)}{\theta^4+6} \right]^i = \sum_{j=0}^{\infty} \binom{i}{j} \left[\frac{\theta x(\theta^2 x^2 + 3\theta x + 6)}{\theta^4+6} \right]^j,$$
$$= \sum_{j=0}^{\infty} \binom{i}{j} \sum_{k=0}^{j} \binom{j}{k} \sum_{l=0}^{k} \binom{k}{l} \frac{\theta^j x^j}{(\theta^4+6)^j} \cdot 3^k \cdot 2^l \cdot \theta^{2j-k-l} \cdot x^{2j-k-l}$$

$\therefore \left[1 - \left(1 + \frac{\theta x(\theta^2 x^2 + 3\theta x + 6)}{\theta^4+6} \right) e^{-\theta x} \right]^{\beta(\alpha-1)} = \sum_{i=0}^{\infty} \binom{\beta(\alpha-1)-1}{i} (-1)^i \sum_{j=0}^{\infty} \binom{i}{j} \sum_{k=0}^{j} \binom{j}{k} \sum_{l=0}^{k} \binom{k}{l} \cdot \frac{\theta^j x^j}{(\theta^4+6)^j} \cdot 3^k \cdot 2^l \cdot \theta^{2j-k-l} \cdot x^{2j-k-l} e^{-i\theta x}$.

Therefore,

$T_R(\beta) =$
$\frac{1}{1-\beta} log \left\{\sum_{i=0}^{\infty}\binom{\beta(\alpha-1)-1}{i}(-1)^i \sum_{j=0}^{\infty}\binom{j}{k} \sum_{k=0}^{j}\binom{j}{k} \sum_{l=0}^{k}\binom{k}{l} \frac{3^k \cdot 2^l \cdot \alpha^\beta \cdot \theta^{5\beta+3j-k-l}}{(\theta^4+6)^{j+\beta}} \int_0^\infty x^{3j-k-l} e^{-\theta x(i+\beta)} dx + \right.$
$\left. \sum_{i=0}^{\infty}\binom{\beta(\alpha-1)-1}{i}(-1)^i \sum_{j=0}^{\infty}\binom{j}{k} \sum_{k=0}^{j}\binom{j}{k} \sum_{l=0}^{k}\binom{k}{l} \frac{3^k \cdot 2^l \cdot \alpha^\beta \cdot \theta^{4\beta+3j-k-l}}{(\theta^4+6)^{j+\beta}} \int_0^\infty x^{3\beta+3j-k-l} e^{-\theta x(i+\beta)} dx \right\}.$

Recall $\int_0^\infty x^n e^{-ax} dx = \frac{\Gamma(n+1)}{a^{n+1}}$ and $\Gamma(\alpha) = (\alpha-1)!$, hence

$T_R(\beta) =$
$\frac{1}{1-\beta} log \left\{\sum_{i=0}^{\infty}\binom{\beta(\alpha-1)-1}{i}(-1)^i \sum_{j=0}^{\infty}\binom{j}{k} \sum_{k=0}^{j}\binom{j}{k} \sum_{l=0}^{k}\binom{k}{l} \frac{3^k \cdot 2^l \cdot \alpha^\beta \cdot \theta^{5\beta+3j-k-l}}{(\theta^4+6)^{j+\beta}} \cdot \theta^{-(3j-k-l+1)} \frac{(3j-k-l)!}{[i+\beta]^{3j-k-l+1}} + \right.$
$\left. \sum_{i=0}^{\infty}\binom{\beta(\alpha-1)-1}{i}(-1)^i \sum_{j=0}^{\infty}\binom{j}{k} \sum_{k=0}^{j}\binom{j}{k} \sum_{l=0}^{k}\binom{k}{l} \frac{3^k \cdot 2^l \cdot \alpha^\beta \cdot \theta^{4\beta+3j-k-l}}{(\theta^4+6)^{j+\beta}} \cdot \theta^{-(3\beta+3j-k-l+1)} \frac{(3\beta+3j-k-l)!}{[i+\beta]^{3\beta+3j-k-l+1}} \right\}.$

$T_R(\beta) = \frac{1}{1-\beta} log \left\{\sum_{i=0}^{\infty}\binom{\beta(\alpha-1)-1}{i}(-1)^i \sum_{j=0}^{\infty}\binom{j}{k} \sum_{k=0}^{j}\binom{j}{k} \sum_{l=0}^{k}\binom{k}{l} \frac{3^k \cdot 2^l \cdot \alpha^\beta \cdot \theta^{5\beta-1}}{(\theta^4+6)^{j+\beta}} \cdot \frac{(3j-k-l)!}{[i+\beta]^{3j-k-l+1}} + \right.$
$\left. \sum_{i=0}^{\infty}\binom{\beta(\alpha-1)-1}{i}(-1)^i \sum_{j=0}^{\infty}\binom{j}{k} \sum_{k=0}^{j}\binom{j}{k} \sum_{l=0}^{k}\binom{k}{l} \frac{3^k \cdot 2^l \cdot \alpha^\beta \cdot \theta^{\beta-1}}{(\theta^4+6)^{j+\beta}} \cdot \frac{(3\beta+3j-k-l)!}{[i+\beta]^{3\beta+3j-k-l+1}} \right\}.$

Let

$$A_{i,j,k,l} = \sum_{i=0}^{\infty}\binom{\beta(\alpha-1)-1}{i}(-1)^i \sum_{j=0}^{\infty}\binom{i}{j} \sum_{k=0}^{j}\binom{j}{k} \sum_{l=0}^{k}\binom{k}{l} \frac{3^k \cdot 2^l \cdot \alpha^\beta \cdot \theta^{5\beta-1}}{(\theta^4+6)^{\beta+j}} \; and \; B_{i,j,k,l}$$
$$= \sum_{i=0}^{\infty}\binom{\beta(\alpha-1)-1}{i}(-1)^i \sum_{j=0}^{\infty}\binom{i}{j} \sum_{k=0}^{j}\binom{j}{k} \sum_{l=0}^{k}\binom{k}{l} \frac{3^k \cdot 2^l \cdot \alpha^\beta \cdot \theta^{\beta-1}}{(\theta^4+6)^{\beta+j}}$$

Therefore,

$$T_R(\beta) = \frac{1}{1-\beta} log \left\{ A_{i,j,k,l} \frac{(3j-k-l)!}{[i+1]^{3j-k-l+1}} + B_{i,j,k,l} \frac{(3\beta+3j-k-l)!}{[i+1]^{3\beta+3j-k-l+1}} \right\}.$$

3.5 Reliability analysis

Given any probability distribution, the reliability analysis is always considered based on the survival function and the hazard rate function of the distribution. Hence, for the extended Pranav distribution, the survival and hazard rate function is given below;

3.5.1 Survival function

The survival function is defined as the probability that an item does not fail prior to some time t. It is given by

$$S(x) = 1 - Q(x) \tag{14}$$

$$S(x) = 1 - \left[1 - \left[1 + \frac{\theta x(\theta^2 x^2 + 3\theta x + 6)}{\theta^4 + 6}\right] e^{-\theta x}\right]^\alpha \tag{15}$$

3.5.2 Hazard rate function

The hazard rate function on the other hand can be seen as the conditional probability of failure, given it has survived to the time t. It is given by

$$h(x) = \frac{q(x)}{1-Q(x)} \tag{16}$$

$$h(x) = \frac{\alpha \frac{\theta^4}{\theta^4+6}(\theta+x^3)\left[1-\left(1+\frac{\theta x(\theta^2x^2+3\theta x+6)}{\theta^4+6}\right)e^{-\theta x}\right]^{\alpha-1} e^{-\theta x}}{1-\left[1-\left[1+\frac{\theta x(\theta^2x^2+3\theta x+6)}{\theta^4+6}\right]e^{-\theta x}\right]^{\alpha}} \tag{17}$$

Figs. 3 and 4 shows the graph of the hazard rate function and survival function of the extended Pranav distribution for various values of the parameters. The hazard rate function graph shows a monotonically increasing shape.

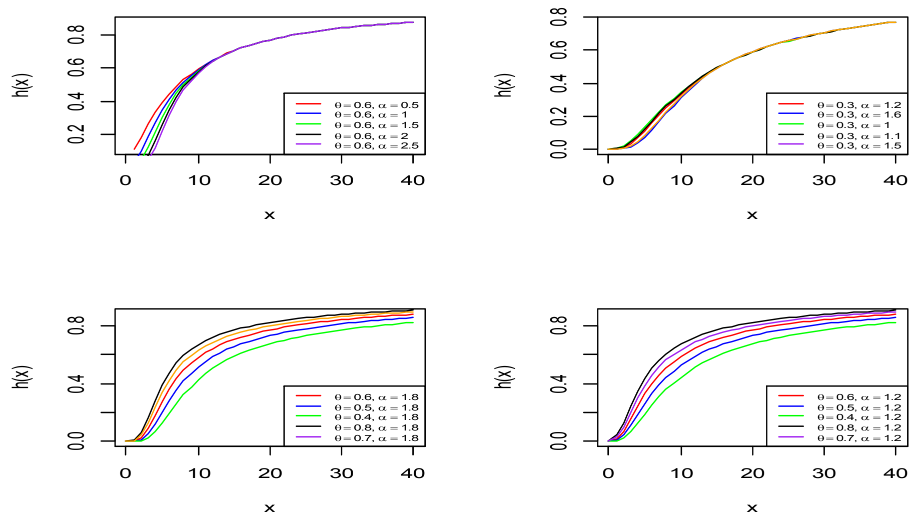

Fig. 3. The Hazard rate function of the extended Pranav distribution

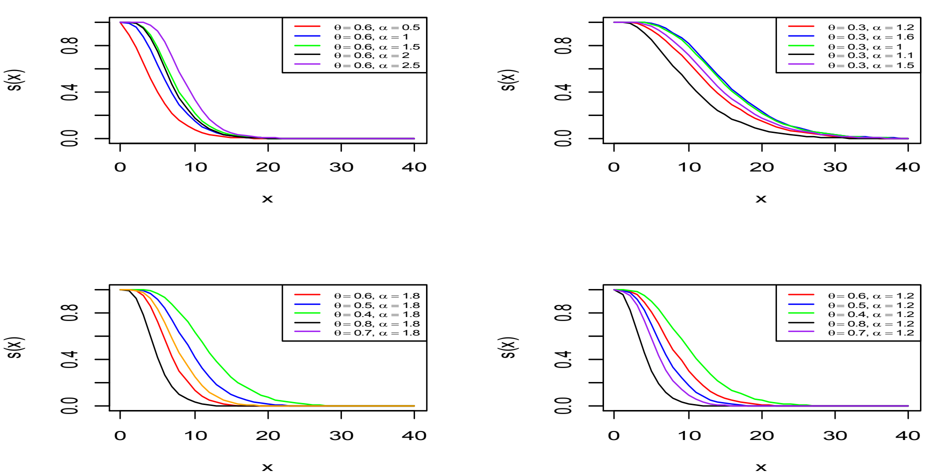

Fig. 4. The survival function of the extended Pranav distribution

4 Maximum Likelihood Estimation

Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the extended Pranav distribution. The log-likelihood function of parameters can be written as

$$LL(\alpha,\theta) = \prod_{i=1}^{n} \ln q(x_i) \tag{18}$$

$$= \prod_{i=1}^{n} \ln\left\{\frac{\alpha\theta^4}{\theta^4+6}(\theta+x_i^3)\left[1-\left(1+\frac{\theta x_i(\theta^2 x_i^3+3\theta x_i+6)}{\theta^4+6}\right)e^{-\theta x_i}\right]^{\alpha-1} e^{-\theta x_i}\right\}$$

$$= \ln\left\{\left(\frac{\alpha\theta^4}{\theta^4+6}\right)^n \prod_{i=1}^{n}(\theta+x_i^3)\, e^{-\theta\sum x_i} \prod_{i=1}^{n}\left[1-\left(1+\frac{\theta x_i(\theta^2 x_i^3+3\theta x_i+6)}{\theta^4+6}\right)e^{-\theta x_i}\right]^{\alpha-1}\right\}$$

$$LL = n[\ln(\alpha)+4\ln(\theta)-\ln(\theta^4+6)] + \sum_{i=1}^{n}\ln(\theta+x_i^3) - \theta\sum_{i=1}^{n} x_i + (\alpha-1)\sum_{i=1}^{n}\ln\big(G_i(\theta)\big) \tag{19}$$

where $G_i(\theta) = \left[1-\left(1+\frac{\theta x_i(\theta^2 x_i^3+3\theta x_i+6)}{\theta^4+6}\right)e^{-\theta x_i}\right]$

In order to maximize the log likelihood, we solve the nonlinear likelihood equations obtained from the differentiation of (19) w.r.t θas shown below;

$$\frac{\partial LL}{\partial\theta} = \partial\left\{n[\ln(\alpha)+4\ln(\theta)-\ln(\theta^4+6)] + \ln(\theta) - \theta\sum_{i=1}^{n} x_i + (\alpha-1)\sum_{i=1}^{n}\ln\big(G_i(\theta)\big)\right\}$$

Let

$$\partial(n[\ln(\alpha)+4\ln(\theta)-\ln(\theta^4+6)]) = \partial[n(4\ln(\theta)-\ln v)];\, where\ v=\theta^4+6$$
$$= \frac{4n}{\theta} - \frac{4n\theta^3}{\theta^4+6} = \frac{24n}{\theta^4+6}$$

Also let

$$\partial\left[\sum_{i=1}^{n}\ln\big(G_i(\theta)\big)\right] = \sum_{i=1}^{n}\frac{1}{\big(G_i(\theta)\big)}\partial\big(G_i(\theta)\big)$$

$$\partial\big(G_i(\theta)\big) = \partial\left[1-\left(1+\frac{\theta x_i(\theta^2 x_i^3+3\theta x_i+6)}{\theta^4+6}\right)e^{-\theta x_i}\right] = \partial\left[1-k\cdot e^{-\theta x_i}\right]$$

Let

$$k = 1+\frac{\theta x_i(\theta^2 x_i^3+3\theta x_i+6)}{\theta^4+6} \therefore \partial k = 0+\partial\frac{u}{v}$$
$$\partial\frac{u}{v} = \frac{\theta x_i(\theta^2 x_i^2+3\theta x_i+6)\cdot 4\theta^3 - (\theta^4+6)(3\theta^2 x_i^3+6\theta x_i^2+36x_i)}{(\theta^4+6)^2}$$
$$= \frac{\theta^2 x_i^2(\theta^4 x_i - 6\theta^3 - 18) + \theta x_i(12\theta^4+18\theta^3-36x_i) - 36}{(\theta^4+6)^2}$$

Hence

$$\partial\big(G_i(\theta)\big) = \frac{\theta^3 x_i^3(\theta^4 x_i + 6\theta^3 + 18) + \theta^2 x_i^2(12\theta^4 - 18\theta^3 - 36x_i) - 36\theta x_i^2}{(\theta^4+6)^2}$$

Therefore,

$$\frac{\partial LL}{\partial \theta} = \frac{24n\theta + (\theta^4 + 6)}{\theta(\theta^4 + 6)} - \sum_{i=1}^{n} x_i + \frac{(\alpha - 1)\sum_{i=1}^{n}\left[\frac{\theta^3 x_i^3(\theta^4 x_i + 6\theta^3 + 18) + \theta^2 x_i^2(12\theta^4 - 18\theta^3 - 36x_i) - 36\theta x_i^2}{(\theta^4+6)^2}\right]}{\left[1 - \left(1 + \frac{\theta x_i(\theta^2 x_i^3 + 3\theta x_i + 6)}{\theta^4 + 6}\right)e^{-\theta x_i}\right]} \tag{20}$$

And also from the differentiation of (19) w.r.t. α, we have

$$\frac{\partial LL}{\partial \alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} \ln\big(G_i(\theta)\big) \tag{21}$$

In order to obtain the estimates of the parameters using the nonlinear equations above, we use the Newton–Raphson method available from the maxLik package in the R software.

5 Application

In this section, we demonstrate the flexibility and superiority of the extended Pranav distribution to some competing lifetime distributions using a real-life data set. In order to achieve this, we compare the goodness of fit of the extended Pranav distribution with similar distributions such as the Lindley distribution (LD) by Lindley [16], the exponentiated Lindley distribution (ELD) by Bakouch et al. [10], exponentiated Exponential distributions (EED) by Gupta and Kundu [12], the Akash distribution (AD) by Shanker [1], the Two-parameter Akash (TPAD) by Shanker and Shukla [5] and the recently proposed Pranav distribution by Shukla [7].

This comparison is done using some measures for testing the goodness of fit of a distribution such as the parameter estimates, the log likelihood, the Akaike Information Criteria (AIC) and the Bayesian Information Criteria (BIC).

The real-life data-set adopted in this study is presented in the Table below.

Data set: This data contains the strength of data of glass of the aircraft window as reported by Fuller et al. [17] as shown below.

18.83	20.8	21.657	23.03	23.23	24.05
24.321	25.5	25.52	25.8	26.69	26.77
26.78	27.05	27.67	29.9	31.11	33.2
33.73	33.76	33.89	34.76	35.75	35.91
36.98	37.08	37.09	39.58	44.045	45.29
45.381					

Table 1 shows that the extended Pranav distribution provides better fit than the competing distribution which confirms its superiority of the other distributions.

Table 1. MLE's, -2ln L, AIC, BIC of the fitted distributions of data sets

Model	Parameter estimates	-2LL	AIC	BIC
AD	0.0971	240.6818	242.6818	244.1157
TPAD	$\hat{\theta} = 3.861 \times 10^{-2}$ $\hat{\lambda} = 1.280 \times 10^{4}$	274.53	278.5301	281.398
EED	$\hat{\theta} = 0.1660$ $\hat{\alpha} = 33.7818$	208.2686	212.6286	215.1366
PD	0.129818	232.77	234.77	236.68
EPD	$\boldsymbol{\hat{\theta} = 0.2423}$ $\boldsymbol{\hat{\alpha} = 9.61\ 46}$	**141.8131**	**145.8131**	**148.681**
LD	0.0630	253.9884	255.9884	257.4224
ELD	$\hat{\theta} = 0.1934$ $\hat{\beta} = 36.6683$	208.2038	212.2038	215.0718

6 Conclusion

This study proposed a new distribution known as the extended Pranav distribution using the widely used exponentiation technique. The mathematical properties of the newly developed distribution including the Order statistics, Entropy, Moments and Moment generating function and reliability analysis was also proposed and derived. Furthermore, the maximum likelihood estimation was discussed. To demonstrate the superiority, the extended Pranav distribution was compared with the Akash, Two-parameter Akash, Lindley and exponentiated distributions among others using a real-life data set. The results obtained easily showed that the extended Pranav outperformed the other distributions.

Competing Interests

Authors have declared that no competing interests exist.

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