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Combined Effects of Hall Current and Magnetic Field on Unsteady Flow Past a Semi-infinite Vertical Plate with Thermal Radiation and Heat Source

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

In the present study combined effects of Hall current and magnetic field on unsteady laminar boundary layer flow of a chemically reacting incompressible viscous fluid along a semi-infinite vertical plate with thermal radiation and heat source is analyzed numerically. A magnetic field of uniform strength is applied normal to the flow. Viscous dissipation and thermal diffusion effects are included. In order to establish a finite boundary condition ($\eta \to 1$) instead of an infinite plate condition, the governing equations in non- dimensional form are transformed to new system of co-ordinates. Obtaining exact solution for this new system of differential equations is very difficult due to its coupled non-linearity, so they are transformed to system of linear equations using implicit finite difference formulae and these are solved using 'Gaussian elimination' method and for this simulation is carried out by coding in C-Program. Graphical results for velocity, temperature and concentration fields are presented and discussed. The results obtained for skin-friction coefficient, Nusselt and Sherwood numbers are discussed and compared with previously published work in the absence of Hall current parameter. These comparisons have shown a good agreement between the results. A research finding of this study, achieved that the velocity and temperature profiles are severely affected by the Hall effect and magnetic field and also a considerable enhancement in

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temperature, main and secondary flow velocities of the fluid is observed for increasing values of radiation parameter.

Keywords: Hall current; magnetic field; radiative heat flux; chemical reaction; implicit finite difference method.

NOMENCLATURE

ρ	*Density*
C_p	*Specific heat at constant pressure*
v	*Kinematic viscosity*
k	*Thermal conductivity*
U	*Mean velocity*
Sc	*Schmidt number*
T	*Temperature*
k_r^2	*Chemical reaction rate constant*
ϵ	*Small reference parameter << 1*
Pr	*Prandtl number*
Gr	*Free convection parameter due to temperature*
Gm	*Free convection parameter due to concentration*
m	*Hall current*
A	*Suction parameter*
n	*A constant exponential index*
D	*Molar diffusivity*
NR	*Thermal radiation parameter*
β	*Coefficient of volumetric thermal expansion of the fluid*
β^*	*Volumetric coefficient of expansion with concentration*
M	*Magnetic parameter*
σ	*Electrical conductivity*
ω_e	*Eectron frequency*
τ_e	*Eectron collision time*
e	*Electron pressure*
n_e	*Number density of the electron*
P_e	*Electron pressure*
So	*Soret number*
Ec	*Viscous dissipation*

1. INTRODUCTION

Considerable attention has been given to the unsteady free-convection flow of viscous incompressible, electrically conducting fluid in the presence of applied magnetic field in connection with the theory of fluid motion in the liquid core of the earth, meteorological and oceanographic applications. Due to the gyration and drift of charged particles, the conductivity parallel to the electric field is reduced and the current is induced in the direction normal to both electric and magnetic fields. This phenomenon is known as the ‘Hall effect’. This effect on the fluid flow with variable concentration has a lot of applications in MHD power generators, general astrophysical and meteorological studies and it can be taken into account within the range of magneto hydro dynamical approximations. Hiroshi Sato [1] has studied the effect of Hall current on the steady hydro magnetic flow between two parallel plates. Masakazu Katagiri [2] studied the steady incompressible boundary layer flow past a semi infinite flat plate in a transverse magnetic field at small magnetic Reynolds number considering with the effect of Hall current. On the other hand Hossain [3] studied the unsteady flow of incompressible fluid along an infinite vertical porous flat plate subjected to suction/injection velocity proportional to $(\text{time})^{-1/2}$. Hossain and Rashid [4] investigated the effect of Hall current on the unsteady free convection flow of a viscous incompressible fluid with mass transfer along a vertical porous plate subjected to a time dependent transpiration velocity when the constant magnetic field is applied normal to the flow. Sri Gopal Agarwal [5] discussed the effect of hall current on the unsteady hydro magnetic flow of viscous stratified fluid through a porous medium in the free convection currents. Ajay Kumar Singh [6] analyzed the steady MHD free convection and mass transfer flow with Hall current, viscous dissipation and joule heating, taking in to account the thermal diffusion effect. In all these studies, the effect of Hall current with radiation on the flow field has not been discussed.

Several authors have dealt with heat flow and mass transfer over a vertical porous plate with variable suction, heat absorption/ generation, radiation and chemical reaction. Actually many process in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for air craft, missiles, satellites and space vehicles are examples of such engineering areas. In such cases one has to take into account the effects of radiation. So, Perdikis and Raptis [7] illustrated the heat transfer of a micro polar fluid in the presence of radiation. Takhar et al. [8] considered the effects of radiation on free-convection flow of a radiation gas past a semi infinite vertical plate in the presence of magnetic field. Raptis and Massalas [9] studied the magneto-hydrodynamic flow past a plate by the presence of radiation. Elbashbeshby and Bazid [10] have reported the effect of radiation on forced convection flow of a micro polar fluid over a horizontal plate. Chamka et al. [11] studied the effect of radiation on free convection flow past a semi infinite vertical plate with mass transfer. Ganeshan and Loganathan [12] analyzed the radiation and mass transfer effects on flow of an incompressible viscous fluid past a moving cylinder. Kim et al. [13] analyzed the effect of radiation on transient mixed convection flow of a micropolar fluid past a moving semi infinite vertical porous plate. Makinde [14] examined the transient free convection interaction with thermal radiation of an absorbing-emitting fluid. Perdikis and Rapti [15] discussed unsteady magnetic hydrodynamic flow in the presence of radiation.

Ramachandra Prasad et al. [16] considered the effects radiation and mass transfer on two dimensional flow past an infinite vertical plate. Chaudhary and Preethi Jain [17] presented an analysis to study the effects of radiation on the hydromagnetic free convection flow of an electrically conducting micropolar fluid past a vertical porous plate through a porous medium in slip-flow regime. The effect of thermal radiation, time-dependent suction and chemical reaction on the two-dimensional flow of an incompressible Boussinesq fluid, applying a perturbation technique has been studied by Prakash and Ogulu [18]. Later, for this same study a numerical investigation is carried out by Rajireddy and Srihari [19]. Ibrahim et al. [20] analyzed the effects of the chemical reaction and radiation absorption on transient hydro-magnetic free-convention flow past a semi infinite vertical permeable moving plate with wall transpiration and heat source. SudheerBabu and Satyanarayana [21] discussed the effects of the chemical reaction and radiation absorption in the presence of magnetic field on free convection flow through porous medium with variable suction. Dulalpal and Babulal Talukdar [22] has made the perturbation analysis to study the effects thermal radiation and chemical reaction on magneto-hydrodynamic unsteady heat and mass transfer in a boundary layer flow past a vertical permeable plate in the slip flow regime. Satyanarayana et al. [23] studied the steady magneto-hydrodynamic free convection viscous incompressible fluid flow past a semi infinite vertical porous plate with mass transfer and hall current. Anand Rao et al. [24] analyzed the effects of viccous dissipation and Soret on an unsteady two-dimensional laminar mixed convective boundary layer flow of a chemically reacting viscous incompressible fluid, along a semi-infinite vertical permeable moving plate. Satyanarayana et al. [25] analyzed the effects of Hall current and radiation absorption on magneto-hydrodynamic free convection flow of a micropolar fluid in a rotating frame of reference. Harish Babu and Satya Narayana [26] discussed the variation of permeability and radiation on the heat and mass transfer flow micropolar fluid along a vertical moving porous plate by considering the effect of transverse magnetic field in to account. In addition to this, Satyanarayana et al. [27] studied the effects of chemical reaction and radiation absorption on magneto-hydrodynamic free-convection flow of a micropolar fluid in a rotating system with heat source. Recently, Srihari and Kesava Reddy [28] have made the numerical investigation to study the effects of soret and magnetic field on unsteady laminar boundary layer flow of a radiating and chemically reacting incompressible viscous fluid along a semi-infinite vertical plate. More recently, Srihari and Srinivas Reddy [29] studied the effects of radiation and soret number variation in the presence of heat source/sink on unsteady laminar boundary layer flow of chemically reacting incompressible viscous fluid along a semi-infinite vertical plate with viscous dissipation.

In most of the earlier studies analytical or perturbation methods were applied to obtain the solution of the problem and there seems to be no significant consideration of the combined effects of Hall current and magnetic field with thermal radiation. Moreover, when the radiative heat transfer takes place, the fluid involved can be electrically conducting in the sense that it is ionized owing to the high operating temperature. Accordingly, it is of interest to examine the effect of magnetic field on the flow and when the strength of applied magetic field is strong, one cannot neglect the effect of Hall current. So in the present study the combined effects of magnetic field and Hall current on unsteady laminar flow of a chemically reacting incompressible viscous fluid along a semi-infinite vertical plate with thermal radiation is investigated. A magnetic field of uniform strength is applied normal to the fluid flow. In order to obtain the approximate solution and to describe the physics of the problem, the present non-linear boundary value problem is solved numerically using implicit finite difference formulae known as Crank-Nicholson method. The obtained results are discussed in detail and compared with the results of Skin-friction, Nusselt and Sher-wood numbers, presented by Srihari and Srinivas Reddy [29] in the absence of Hall current parameter.

2. FORMULATION OF THE PROBLEM

An unsteady laminar, boundary layer flow of a viscous, incompressible, electrically conducting dissipative and chemically reacting fluid along a semi-infinite vertical plate, with thermal radiation, heat source is considered. The x'-axis taken along the plate in the vertically upward direction and y'-axis normal to it as shown in the Fig. 2.1. A magnetic field of uniform strength applied along y'-axis. Further, due to the semi-infinite plane surface assumption, the flow variables are functions of normal distance y' and t' only. A time dependent suction velocity is assumed normal to the plate. A magnetic field of uniform strength is assumed to be applied transversely to the porous plate. The magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field can be neglected. The equation of conservation of electric charge $\nabla . \vec{J} = 0, gives \quad j_y$ = constant, where $\vec{J} = (j_x, j_y, j_z)$.We further assume that the plate is non-conducting. This implies $j_y = 0$ at the plate and hence zero everywhere. When the strength of magnetic field is very large the generalized Ohm's law, in the absence of electric field takes the following form:

$$\vec{J} + \frac{\omega_e \tau_e}{B_0}(\vec{J} \times \vec{B}) = \sigma\left(\vec{V} \times \vec{B} + \frac{1}{en_e} \nabla P_e\right) \quad (1)$$

Where V is the velocity vector, σ is the electric conductivity, ω_e is the electron frequency, τ_e is the electron collision time, e is the electron charge, n_e is the number density of the electron and P_e is the electron pressure. Under the assumption that the electron pressure (for weakly ionized gas), the thermo-electric pressure and ion-slip are negligible, equation (2.1) becomes:

$$J_x = \frac{\sigma B_0}{1 + m^2}(mu - w) \text{ and}$$

$$j_z = \frac{\sigma B_0}{1 + m^2}(u + mw) \quad (2)$$

where u is the x-component of V, w is the z component of V and $m\,(= w_e \tau_e)$ is the Hall parameter.

Within the above framework, the equations which govern the flow under the usual Boussinesq approximation are as follows:

- **Continuity**

$$\frac{\partial v'}{\partial y'} = 0 \tag{3}$$

- **Momentum equations**

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \upsilon\frac{\partial^2 u'}{\partial y'^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2}{\rho(1+m^2)}(u' + mw') \tag{4}$$

$$\frac{\partial w'}{\partial t'} + v'\frac{\partial w'}{\partial y'} = \upsilon\frac{\partial^2 w'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho(1+m^2)}(w' - mu') \tag{5}$$

- **Energy**

$$\frac{\partial T}{\partial t'} + v'\frac{\partial T}{\partial y'} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y'^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y'} + \frac{\mu}{\rho c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{Q(T - T_\infty)}{\rho c_p} \tag{6}$$

- **Mass transfer**

$$\frac{\partial C}{\partial t'} + v'\frac{\partial C}{\partial y'} = D\frac{\partial^2 C}{\partial y'^2} + \frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y'^2} - k_r^2 C \tag{7}$$

The radiative flux q_r by using the Rosseland approximation [30], is given by

$$q_r = -\frac{4\sigma^*}{3a_R}\frac{\partial T^4}{\partial y'} \tag{8}$$

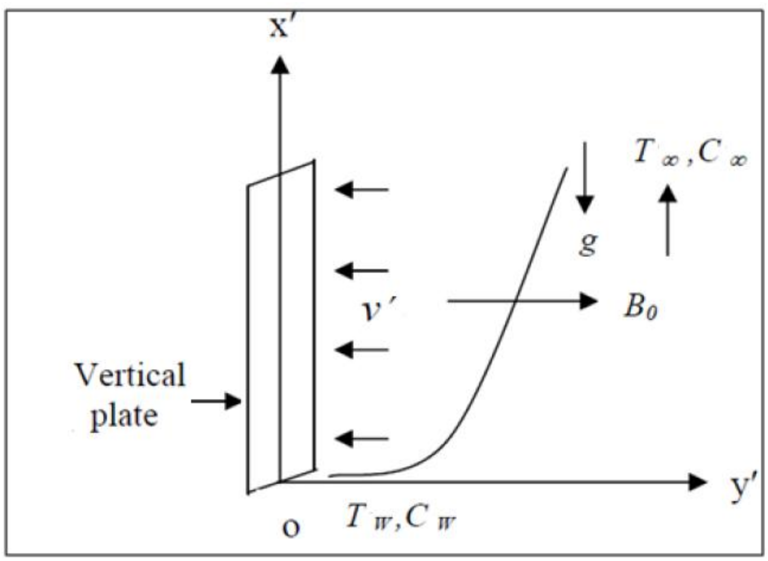

Fig. 2.1. Schematic diagram of flow geometry

The boundary conditions suggested by the physics of the problem are

$$u' = U_0,\ w' = 0,\ T = T_W + \varepsilon(T_W - T_\infty)e^{n't'},\ C = C_W + \varepsilon(C_W - C_\infty)e^{n't'} \quad at\ \ y' = 0$$

$$u' \to 0,\ w' = 0,\ T \to T_\infty,\quad C \to C_\infty \qquad as\ \ y' \to \infty \tag{9}$$

It has been assumed that the temperature differences within the flow are sufficiently small and T^4 may be expressed as a linear function of the temperature *T* using Taylor series as follows

Let the Taylor series about T_∞, be $T^4 = T_\infty^4 + 4(T-T_\infty)T_\infty^3 + 12\frac{(T-T_\infty)^2}{2!}T_\infty^2 + \dots$,

Neglecting the higher order terms in the above series, we have

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \tag{10}$$

Using (10) in (8) and then (8) in (6), it implies

$$\frac{\partial T}{\partial t'} + v'\frac{\partial T}{\partial y'} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y'^2} - \frac{16\sigma^* T_\infty^3}{3\rho c_p k^*}\frac{\partial^2 T}{\partial y'^2} + \frac{\mu}{\rho c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{Q(T-T_\infty)}{\rho c_p} \tag{11}$$

Integration of continuity eqn (1) for variable suction velocity normal to the plate gives

$$v' = -U_0\left(1+\varepsilon A e^{n't'}\right) \tag{12}$$

where *A* is the suction parameter and εA is less than unity. Here U_0 is mean suction velocity, which is a non-zero positive constant and the minus sign indicates that the suction is towards the plate.

In order to obtain the non-dimensional partial differential equations with boundary conditions, introducing the following non-dimensional quantities,

$$\begin{gathered} u = \frac{u'}{U_0},\ w = \frac{w'}{U_0},\ y = \frac{y'U_0}{\upsilon}\quad t = \frac{U_0^2 t'}{\upsilon},\ \theta = \frac{T-T_\infty}{T_w - T_\infty}, \\ \phi = \frac{C-C_\infty}{C_w - C_\infty},\ \Pr = \frac{\mu C_p}{k},\ Sc = \frac{\upsilon}{D},\ M = \frac{\sigma B_0^2 \upsilon}{\rho U_0^2},\ So = \frac{D_m k_T (T_w - T_\infty)}{\upsilon T_m (C_w - C_\infty)} \\ Gr = \frac{g\beta\upsilon(T_w - T_\infty)}{U_0^3},\ Gm = \frac{g\beta^*\upsilon(C_w - C_\infty)}{U_0^3},\ S = \frac{Q\upsilon}{\rho C_p U_0^2} \end{gathered} \tag{13}$$

$Kr = \frac{k_r'^2 \upsilon}{U_0^2},\ NR = \frac{16\sigma^* T_\infty^3}{3ka_R},\ Ec = \frac{U_0^2}{C_p(T_w - T_\infty)}\quad n = \frac{\upsilon n'}{U_0^2}$, in to equations (4), (5), (7) and (11), we get

$$\frac{\partial u}{\partial t} - \left(1+\varepsilon A e^{nt}\right)\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - \frac{M}{1+m^2}(u+mw) + Gr\theta + Gm\phi \tag{14}$$

$$\frac{\partial w}{\partial t} - \left(1+\varepsilon A e^{nt}\right)\frac{\partial w}{\partial y} = \frac{\partial^2 w}{\partial y^2} - \frac{M}{1+m^2}(w-mu) \tag{15}$$

$$\frac{\partial \theta}{\partial t} - \left(1+\varepsilon A e^{nt}\right)\frac{\partial \theta}{\partial y} = \left(\frac{1+NR}{Pr}\right)\frac{\partial^2 \theta}{\partial y^2} + Ec\left(\frac{\partial u}{\partial y}\right)^2 + S\theta \tag{16}$$

$$\frac{\partial \phi}{\partial t} - \left(1 + \varepsilon A e^{nt}\right)\frac{\partial \phi}{\partial y} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} + So\frac{\partial^2 \theta}{\partial y^2} - Kr\phi \tag{17}$$

with the boundary conditions

$$\begin{aligned} &u = 1,\ w = 0,\ \ \theta = 1 + \varepsilon e^{nt},\ \ \phi = 1 + \varepsilon e^{nt} \quad \text{at } y = 0 \\ &u \to 0,\ w = 0,\ \ \theta \to 0,\ \ \phi \to 0 \quad \text{as } y \to \infty \end{aligned} \tag{18}$$

In order to establish a finite plate condition, $\eta \to 1$ in equation (18) instead of an infinite boundary condition, $y \to \infty$, employing the transformation $\eta = 1 - e^{-y}$ on equations (14)-(18), we get

$$\frac{\partial u}{\partial t} - \left(1 + \varepsilon A e^{nt}\right)(1-\eta)\frac{\partial u}{\partial \eta} = (1-\eta)^2\frac{\partial^2 u}{\partial \eta^2} - (1-\eta)\frac{\partial u}{\partial \eta} - \frac{M}{1+m^2}\left(u + mw\right) + Gr\theta + Gm\phi \tag{19}$$

$$\frac{\partial w}{\partial t} - \left(1 + \varepsilon A e^{nt}\right)(1-\eta)\frac{\partial w}{\partial \eta} = (1-\eta)^2\frac{\partial^2 w}{\partial \eta^2} - (1-\eta)\frac{\partial w}{\partial \eta} - \frac{M}{1+m^2}\left(w - mu\right) \tag{20}$$

$$\frac{\partial \theta}{\partial t} - \left(1 + \varepsilon A e^{nt}\right)(1-\eta)\frac{\partial \theta}{\partial \eta} = \left(\frac{1+NR}{Pr}\right)\left((1-\eta)^2\frac{\partial^2 \theta}{\partial \eta^2} - (1-\eta)\frac{\partial \theta}{\partial \eta}\right) + Ec\left((1-\eta)\frac{\partial u}{\partial \eta}\right)^2 + S\theta \tag{21}$$

$$\begin{aligned} \frac{\partial \phi}{\partial t} - \left(1 + \varepsilon A e^{nt}\right)(1-\eta)\frac{\partial \phi}{\partial \eta} = \frac{1}{Sc}&\left((1-\eta)^2\frac{\partial^2 \phi}{\partial \eta^2} - (1-\eta)\frac{\partial \phi}{\partial \eta}\right) + \\ &So\left((1-\eta)^2\frac{\partial^2 \theta}{\partial \eta^2} - (1-\eta)\frac{\partial \theta}{\partial \eta}\right) - Kr\phi \end{aligned} \tag{22}$$

with boundary conditions

$$\begin{aligned} &u = 1:\ \ w = 0,\ \theta = 1 + \varepsilon e^{nt},\ \ \phi = 1 + \varepsilon e^{nt} \quad at\ \ \eta = 0 \\ &u \to 0: w = 0,\ \theta \to 0,\ \ \ \theta \to 1 + \varepsilon e^{nt} \quad as\ \ \eta \to 1 \end{aligned} \tag{23}$$

3. METHOD OF SOLUTION

Equations (19)-(22) are non-linear coupled, differential equations, for which obtaining exact solution is very difficult, so they are transformed to system of linear equations using implicit finite difference formulae, as follows

$$-P_3\, r\, u_{i-1}^{j+1} + \left(1 + 2P_3\, r\right)u_i^{j+1} - P_3\, r\, u_{i+1}^{j+1} = E_i^j \tag{24}$$

$$-P_3\, r\, w_{i-1}^{j+1} + \left(1 + 2P_3\, r\right)w_i^{j+1} - P_3\, r\, w_{i+1}^{j+1} = D_i^j \tag{25}$$

$$-P_3 P_4\, r\, \theta_{i-1}^{j+1} + \left(1 + 2P_3 P_4\, r\right)\theta_i^{j+1} - P_3 P_4\, r\, \theta_{i+1}^{j+1} = F_i^j \tag{26}$$

$$-\frac{P_3 r}{Sc}\phi_{i-1}^{j+1} + \left(1+\frac{2P_3 r}{Sc}\right)\phi_i^{j+1} - \frac{P_3 r}{Sc}\phi_{i+1}^{j+1} = H_i^j \tag{27}$$

with boundary conditions in finite difference form

$$\begin{aligned} &u(0,j)=1, \quad \theta(0,j)=1+\varepsilon\exp(n.j.k_1), \quad \phi=1+\varepsilon\exp(n.j.k_1), \quad \forall j \\ &u(10,j)\to 0, \quad \theta(10,j)\to 0, \quad \phi(10,j)\to 1 \quad \forall j \end{aligned} \tag{28}$$

where

$$E_i^j = P_3 r\, u_{i-1}^j - \left(1 - P_1P_2\, rh - 2P_3 r + P_2 rh - \frac{M\, m}{1+m^2}k_1\right)u_i^j + \left(P_1\, P_2 rh + P_3 r - P_2 rh\right)u_{i+1}^j$$
$$+ Gr\, k_1\theta_i^j + Gm\, k_1\phi_i^j - \frac{M\, m}{1+m^2}k_1\, w_i^j$$

$$D_i^j = P_3 r\, w_{i-1}^j - \left(1 - P_1P_2\, rh - 2P_3 r + P_2 rh - \frac{M\, m}{1+m^2}k_1\right)w_i^j + \left(P_1\, P_2 rh + P_3 r - P_2 rh\right)w_{i+1}^j$$
$$+ \frac{M\, m}{1+m^2}k_1\, u_i^j$$

$$F_i^j = P_3P_4 r\,\theta_{i-1}^j + \left(1 - P_1P_2 rh - 2P_3P_4 r + P_2P_4 rh\right)\theta_i^j + \left(P_1P_2 rh + P_3P_4 r - P_2P_4 rh\right)\theta_{i+1}^j$$

$$H_i^j = \frac{P_3 r}{Sc}\,\phi_{i-1}^j + \left(1 + P_1P_2\, rh - \frac{2P_3 r}{Sc} + \frac{P_2 rh}{Sc} - k_r^2 k_1\right)\phi_i^j + \left(\frac{P_3 r}{Sc} - P_1\, P_2 rh - \frac{P_2 rh}{Sc}\right)\phi_{i+1}^j$$
$$+ \left(2P_3 rS_0 - S_0P_1 rh\right)\theta_{i+1}^j + \left(S_0P_1 rh - 4P_3 rS_0\right)\theta_i^j + 2P_3 rS_0\theta_{i-1}^j$$

$$P_1 = 1 + \in Ae^{nt},\ P_2 = 1 - i\,h,\ P_3 = \frac{(1-i\,h)^2}{2},\ P_4 = \frac{1+NR}{\Pr},$$

where $r = k_1/h^2$ and *h,* k_1 are mesh sizes along η and time direction respectively. Index i refers to space and *j* for time.

To obtain the difference equations, the region of the flow is divided into a grid or mesh of lines parallel to η and t axes. Solutions of difference equations are obtained at the intersection of these mesh lines called nodes. The finite-difference equations at every internal nodal point on a particular *n*-level constitute a tri-diagonal system of equations. These equations are solved by Gaussian elimination method and for this a numerical code is executed using C-Program to obtain the approximate solution of the system. In order to prove the convergence of present numerical scheme, the computation is carried out by slightly changed values of *h,* and k_1, and the iterations on until a tolerance 10^{-8} is attained. No significant change was observed in the values of u, w, θ *and* ϕ. Thus, it is concluded that the finite difference scheme is convergent and stable.

3.1 Skin-friction

The Skin friction coefficient τ is given by

$$\tau = \left.\frac{\partial u}{\partial y}\right|_{y=0} = (1-\eta)\left.\frac{\partial u}{\partial \eta}\right|_{\eta=0}, \tag{29}$$

3.2 Nusselt Number

The rate of heat transfer in terms of Nusselt number is given by

$$Nu = \left.\frac{\partial \theta}{\partial y}\right|_{y=0} = (1-\eta)\left.\frac{\partial \theta}{\partial \eta}\right|_{y=0} \tag{30}$$

3.3 Sherwood Number

The coefficient of Mass transfer which is generally known as Sherwood number, Sh, is given by

$$Sh = \left.\frac{\partial \phi}{\partial y}\right|_{y=0} = (1-\eta)\left.\frac{\partial \phi}{\partial \eta}\right|_{\eta=0} \tag{31}$$

4. RESULTS AND DISCUSSION

In order to obtain the approximate solution and to describe the physics of the problem, in the present work, numerical solution is obtained to study the influence of various flow parameters encountered in the momentum, energy and mass transfer equations. To be realistic, the values of Prandtl number (Pr) are chosen to be Pr = 0.71 and Pr = 7.0, which represent air and water at temperature 20ºC and one atmosphere pressure, respectively.

Figs. (1) and (2) show the effect of Hall current (m) on velocity field's u and w respectively, in the presence of heat source. It is observed that the effect of increasing values of m results in increasing both the velocity profiles u and w. This due to the fact that an increase in hall current produces a deflection on moving fluid so that the level of cross flow velocity is maximum and therefore the fluid is dragged with more velocity. Furthermore, it is noted that both the velocities u and w increase in the presence of heat source as the internal heat generation is to increase the rate of heat transport to the fluid. From Fig. (3), it is interesting to note that there is a considerable enhancement in the secondary flow velocity of the fluid is observed for slightly increasing values of Hall parameter.

From Figs. (4), (5) and (6), it is seen that for increasing values of NR, there is rise in the temperature, main and cross flow velocities. This due to the fact that an increase in the value of radiation parameter $NR = 16\sigma^{*}T_{\infty}^{3}/3k\,a_R$, forgiven k and T_{∞}, leads to decrease in the Roseland radiation absorbtivity (a_R). According to the equations (6) and (8), it is concluded that, the divergence of the radiation heat flux ($\partial q_r / \partial y^{*}$) increases as a_R decreases and it implies that the rate of radiative heat, transferred to the fluid increases and consequently the fluid temperature and therefore main and secondary flow velocities of their particles also increase. Furthermore, it is interested note that velocity u increases in the presence of radiation.

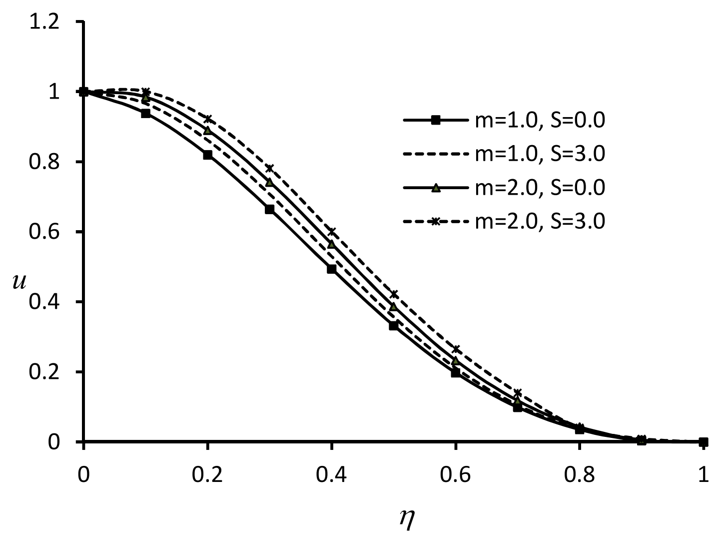

Fig. 1. Effect of Hall current (m) on velocity field u in the presence of heat source
(Gr=5.0, Gm=5.0, M=1.0, Du=1.0, Pr=0.71, Ec=0.5, NR=0.5, Ch=0.5, So=1.0, Sc=0.22, A=0.3 and ε =0.01)

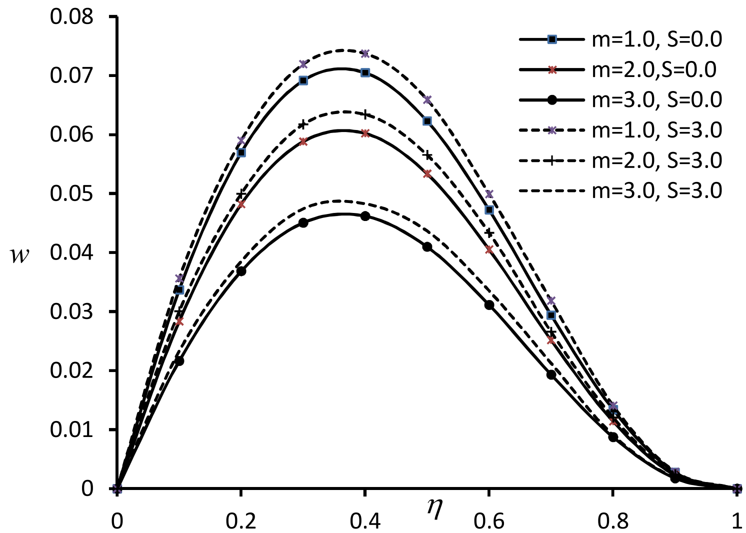

Fig. 2. Effect of Hall current (m) on velocity field w in the presence of heat source
(Gr=5.0, Gm=5.0, M=1.0, So=1.0, Du=1.0, Pr=0.71, Ec=0.5, NR=0.5, Ch=0.5, Sc=0.22, A=0.3 and =0.01)

Figs. (7) and (8) show the effect magnetic parameter M on main and cross flow velocity profiles respectively. It is observed from Fig. (7) that an increase in M leads to decrease in the velocity. This due to the fact that the introduction of transverse magnetic field in an electrically conducting fluid has a tendency to give rise to a resistive-type force called the Lorentz force, which acts against the fluid flow and hence results in retarding the velocity profile. Furthermore, from Fig. (8) it is seen that for increasing values magnetic parameter M, there is a considerable enhancement in the cross flow velocity w, as the impact of deflecting force due to the applied magnetic field on the fluid is predominant.

The effect Prandtl number in the presence of heat source parameter on temperature distribution is shown in Fig. (9). It is evident from figure that the temperature increases in the presence of heat source parameter as the effect of internal heat generation is to increase the rate of heat transport to the fluid. Furthermore it is interesting to note that with increasing values of Prandtl number Pr, there is a decrease in the temperature profile. This due to the physical fact that an increase in Pr leads to decrease in the thermal boundary layer thickness.

Fig (10) shows the species concentration for different gases like Hydrogen (H2: Sc=0.22), Oxygen (O2: Sc=0.66), Ammonia (NH3: Sc=0.78) and S_c = 2.62 for propyl benzene at 20ºC and one atmospheric pressure and for different Kr. It is observed that the effect of increasing values of chemical reaction parameter and Schmidth number is to decrease concentration distribution in the flow region.

Results for Skin-friction coefficient, Nusselt and Sherwood numbers are presented in Tables (1), (2) and (3) respectively, in the presence and absence of Hall effect. A comparative numerical study between present and previous results in tables reveals that Skin–friction, Nusselt number increase in the presence of Hall current parameter but Sherwood number decreases slightly in the presence of Hall effect. Further, it is noted that Skin-friction increases with increasing values of m, NR, Ec, So, Gr and Gm while it decreases for the increasing values of M, Pr. An increase in Ec, m, S leads to an increase in the Nusselt number. For increasing values of Sc and Ch decreases the Sherwood number. But it increases with the increasing values So.

In order to access the validity of the present numerical scheme, the present results are compared with previous published data [29] for Skin-friction, rate of heat and mass transfer in the absence of Hall effect. The comparisons in all the cases are found to be in very good agreement and it gives an indication of high degree of coincidence with realistic physical phenomenon.

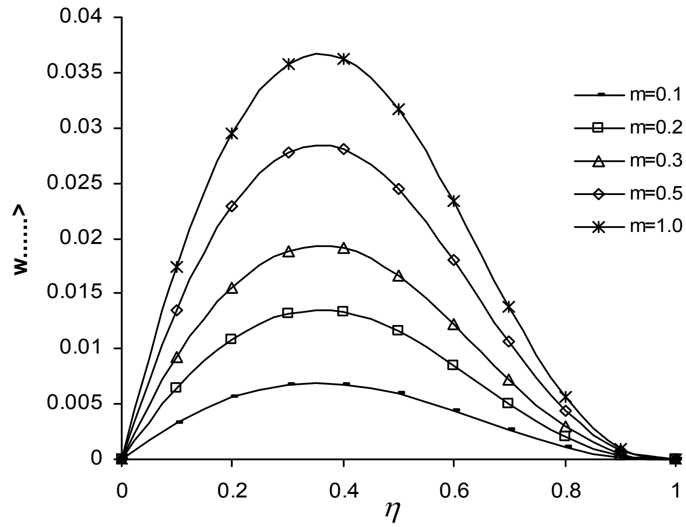

Fig. 3. Effect of Hall current (m) on velocity component w

(Gr=5.0, Gm=5.0, NR=0.5, Ec=0.2, S=0.5, Pr=0.71, Sc=0.22, Kr=0.5, A=0.3, ε =0.01 and t=1.0

Table 1. Effects of Gr, Gm, Pr, Sc, Kr, NR, So and M on Skin-Friction coefficient

Gr	Gm	Pr	Sc	Kr	*NR*	*So*	M	τ S=2.0,Ec=0.5 Previous [29] (m=0.0)	τ S=2.0,Ec=0.5 Present (m=1.0)
5.0	5.0	0.71	0.24	0.5	0.5	0.0	0.0	1.4032	1.4032
5.0	5.0	0.71	0.24	0.5	0.5	0.0	2.0	0.7413	0.98796
5.0	5.0	0.71	0.24	0.5	0.5	2.0	2.0	0.9721	1.17426
5.0	5.0	0.71	0.24	0.5	1.0	2.0	2.0	1.0523	1.24643
5.0	5.0	0.71	0.6	0.5	0.5	2.0	2.0	0.8423	1.01633
5.0	5.0	7.0	0.24	0.5	0.5	2.0	2.0	0.3838	0.68666
5.0	10.0	0.71	0.24	0.5	0.5	2.0	2.0	2.7352	2.97588
10.0	5.0	0.71	0.24	0.5	0.5	2.0	2.0	2.3597	2.58178

Table 2. Effects of NR and Pr on Nusselt – number

NR	*Pr*	*Nu* S=2.0, Ec=0.5 previous [29] (m=0.0)	*Nu* S=2.0, Ec=0.5 present (m=1.0)
0.0	0.71	-1.0807	-0.93922
0.5	0.71	-0.8230	-0.72087
0.5	7.0	-3.6770	-3.12927
0.5	11.4	-4.7594	-4.03651

Table 3. Effects of Sc, Kr and So on Sherwood number

Sc	Kr	So	*Sh* S=2.0, Ec=0.5 previous [29] (m=0.0)	*Sh* S=2.0, Ec=0.5 present (m=1.0)
0.24	0.5	0.0	-0.59393	-0.59393
0.24	0.5	2.0	-0.37159	-0.37652
0.24	1.0	2.0	-0.43987	-0.44012
0.6	0.5	2.0	-0.55924	-0.56102

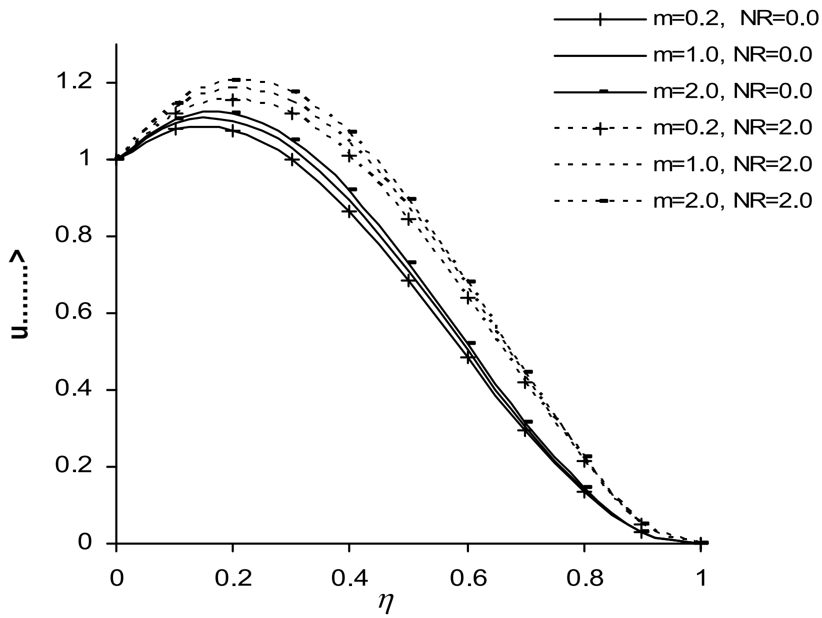

Fig. 4. Effect of Hall current on velocity field u in the presence/absence of radiation
(Gr=5.0, Gm=5.0, M=1.0, Ec=0.2, S=0.5, Pr=0.71, Sc=0.22, Kr=0.5, A=0.3, ε =0.01 and t=1.0)

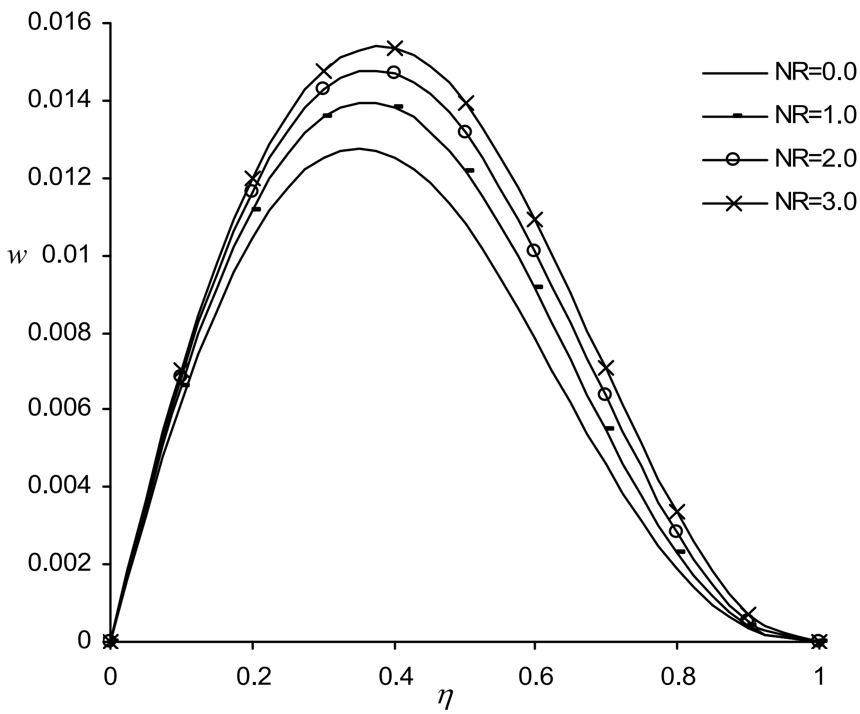

Fig. 5. Effect of radiation (NR) on velocity component w
(Gr=5.0, Gm=5.0, M=1.0, m=0.2, Ec=0.2, S=0.5, Pr=0.71, Sc=0.22, Kr=0.5, A=0.3, ε =0.01 and t=1.0)

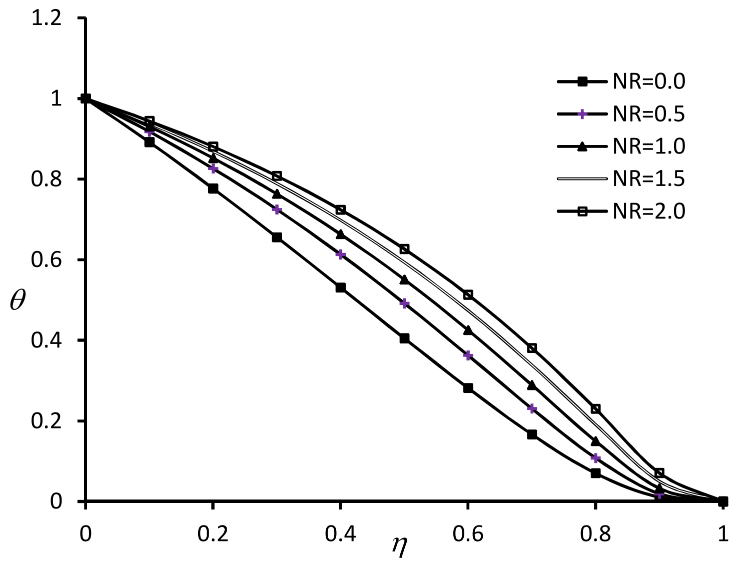

Fig. 6. Effect of radiation (NR) on temperature field (θ)
(Gr=5.0, Gm=5.0, m=1.0, M=1.0, Du=1.0, So=1.0, Pr=0.71, Ec=0.5, S=0.5, Ch=0.5, Sc=0.22, A=0.3 and ε =0.01)

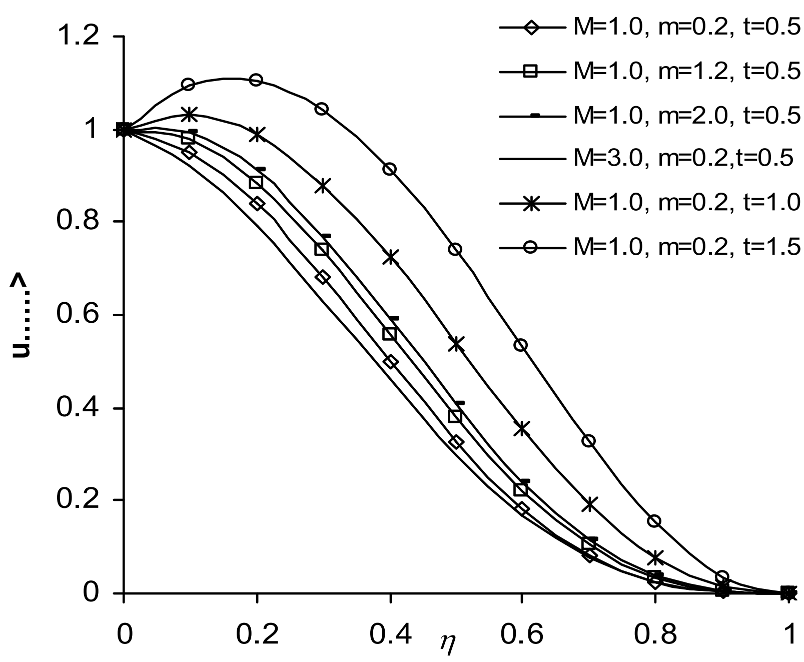

Fig. 7. Effects of magnetic field (M) and Hall current m on velocity field u
(Gr=5.0, Gm=5.0, NR=0.5, Ec=0.2, S=0.5, Pr=0.71, Sc=0.22, Kr=0.5, A=0.3 and ε =0.01)

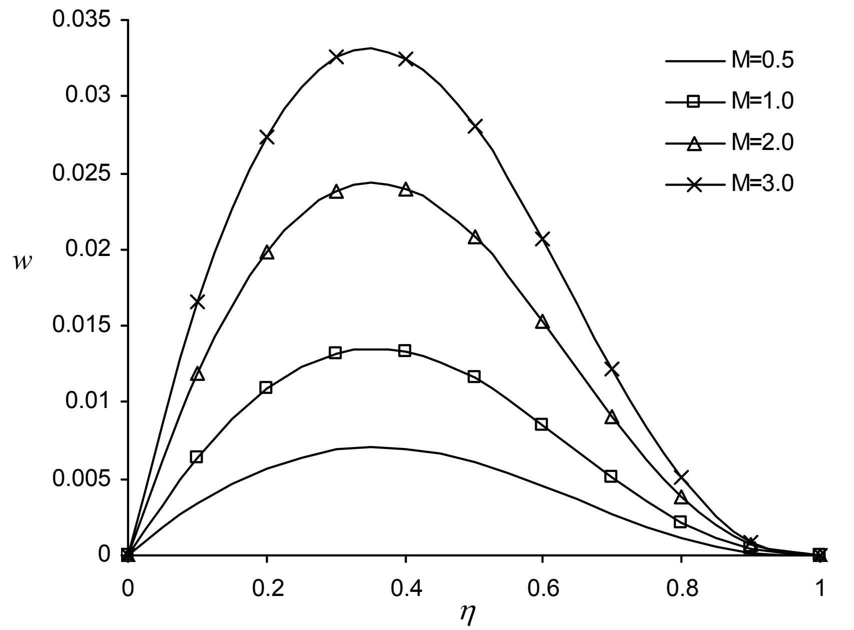

Fig. 8. Effects of magnetic field (M) on velocity component w
(Gr=5.0, Gm=5.0, NR=0.5, Ec=0.2, S=0.5, Pr=0.71, Sc=0.22, Kr=0.5, A=0.3, ε =0.01 and t=1.0

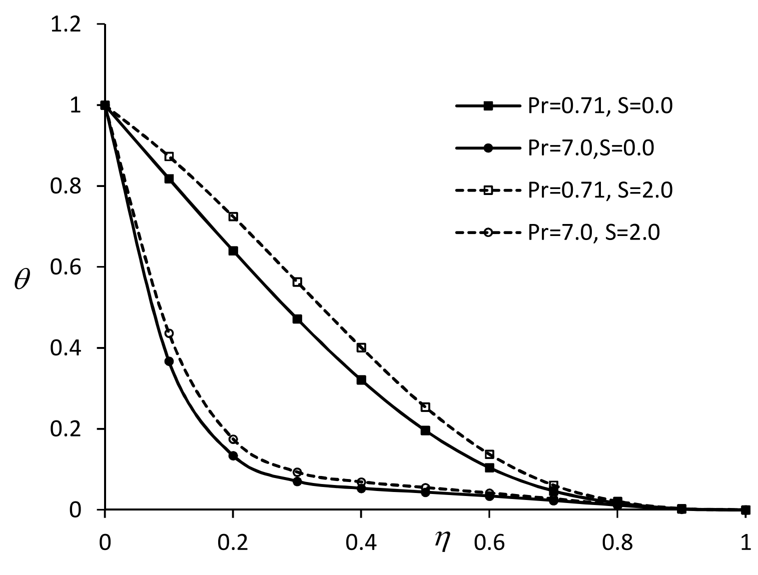

Fig. 9. Effect of Prandtl number (Pr) on temperature field (θ) in the presence of heat source
(Gr=5.0, Gm=5.0, m=1.0, M=1.0, Du=1.0, So=1.0, Ec=0.5, NR=0.5, Ch=0.5, Sc=0.22, A=0.3 and =0.01)
(So=1.0, Du=1.0, Sc=0.22, A=0.3 and =0.01)

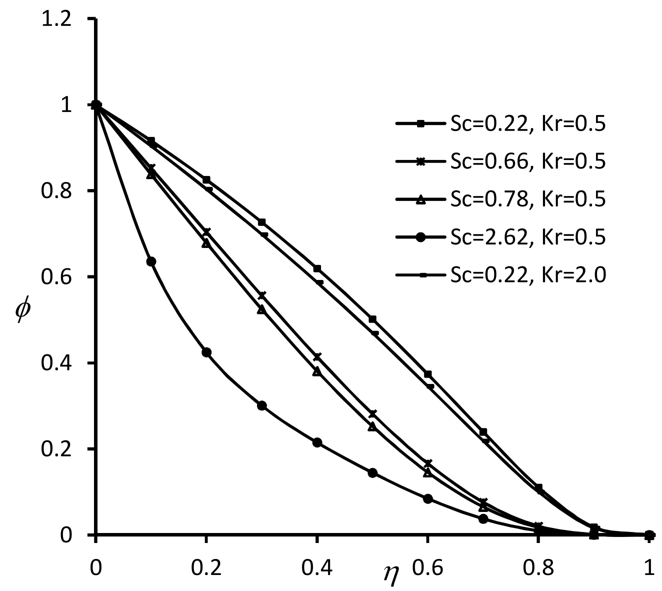

Fig. 10. Effect of Schmidt number and chemical reaction on concentration field
(NR=0.5, Pr=0.71, ε=0.01, n=0.1, A=0.3 and t=1.0)

5. CONCLUSION

Combined effects of Hall current and Magnetic field on unsteady laminar flow of a radiating fluid along a semi-infinite vertical plate, with heat source, viscous dissipation and thermal diffusion are analysed. From this study the following conclusions are drawn.

1. The velocity and temperature profiles are severely affected by the magnetic field and Hall effects.
2. For increasing values of Hall current parameters, there is a considerable enhancement in main and secondary flow velocities of the fluid.
3. Magnetic field reduces the main flow velocity profile but there is a considerable enhancement in the cross flow velocity is observed for increasing values same magnetic parameter M.
4. Skin–friction, Nusselt increase in the presence of Hall effect. The temperature, velocity, Skin–friction and Nusselt number increase in the presence heat source
5. There is a rise in the temperature, primary and secondary velocities of the fluid flow for increasing values of radiation parameter.
6. The comparative study, between present and previously published results [29] for Skin–friction, Nusselt and Sherwood numbers in the absence of Hall parameter, shows a good agreement. And therefore it is concluded that the proposed numerical technique, present in the paper is an efficient algorithm with assured convergence.

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This column is not required as the whole research work was done by me

COMPETING INTERESTS

Author has declared that no competing interests exist.

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