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New Model for the COVID-19 Reported Cases and Deaths of Ghana in Accelerated Spread and Prediction of the Delayed Phase

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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ABSTRACT

There is an ongoing investigation on the transmission characteristics of COVID-19 with respect to country-based inflection points, nature of distribution and prediction of future trends. In this study, a new accelerated and delayed spread models for COVID-19 reported cases and deaths in Ghana were developed. Optimization techniques coupled with interpolations, least square and non-linear regression methods, to come out with an informed modeling strategy to predict the delayed spread for the case of Ghana were adopted. Derivative and tangent methods were also applied to determine inflection points for Ghana's cases and death from COVID-19. The data used for the study covered the first 250 days of events and interventions of the pandemic in Ghana. It was realized that the distribution of the COVID-19 situation in Ghana followed an exponential distribution curve. A modification of the developed model to help optimize the error between observed and estimated values yielded an improvement in the prediction of the delayed phase. Our derived parameters revealed that transmission of the virus between phases depended on changes in the precautionary measures and peoples' behaviors. The study thus shows that Ghana passed her inflection point of reported cases on Sunday 19th July, 2020 and may currently be in the delayed phase characterized with a staggering trend where new infections similar in magnitude to previous infections may upsurge. The correlation between reported cases and deaths revealed linear dependence with positive deviation between accelerated and delayed phases. In conclusion, the study predicted the commencement of a new wave in Ghana after Wednesday October 28, 2020 with higher intensity than what was previously observed if timely impositions of interventions to minimize the effect of the second wave are not taken.

Keywords: COVID-19; empirical modeling; causal correlation; accelerated spread; delayed spread; mortality; Ghana Case

1. INTRODUCTION

The emerging crisis of COVID-19 pandemic continues to have a devastating effect on the world population. The uncertainties regarding the factors, which cause it to accelerate, delay in its spread, and remain dormant in the latent period, require much more attention in research [1]. The latent period (1-14 days) of COVID-19 is a crucial stage for diagnosis, determination of route of transmission of pathogen and the immune system functionality of the host [2]. Several attempts have been made in scientific literature to model the stages of transmission of COVID-19 to reveal some information to help containment.

Bharat [3] classified four stages of COVID-19 transmission. The first stage was classified as imported cases where the confirmed cases have foreign origin and not accompanied by any local spread. In second stage, local transmission occurs at this stage, but the primary source and the people infected can be located and identified. The third stage is characterized with community spread where the source and infected individuals could not be traced and identified, and the final stage is the most overwhelming where there is within country spread and with deaths in different clusters in a form of epidemic, which is difficult to control. WHO [4] outlined the severity of COVID-19 cases into three transmission dynamics; the community level (associated with high spread), cluster level (moderate spread) and sporadic level (associated with low or no cases at all).

Existing literature on COVID-19 has shown some attempts to quantify the country-based transmission risk of COVID-19 using epidemiological models [5-10,11-14]. These authors argued that the current understanding of the transmission of COVID-19 is inadequate since a proper study into its transmission can inform current and future pandemic response. Using epidemiological modeling and Bayesian phylogeographic inference they classified the transmission of the disease in two phases [15,16]. The first phase which occurred in mid-to-late January 2020was interrupted by containment measures in China. The second phase was identified to have occurred extended from late February to mid-March and the most predominant as it was related to unrestricted movements between countries outside of China facilitating intercontinental spread, with Europe being the most affected continent) [17-19]. Hellewell et al. [20] on the other hand predicted the pattern of the virus and concluded that further spread of COVID-19 will be delayed for a period of three months with the interventions of contact tracing and case isolation. However, there was a decrease in the probability of controlling the virus when the time taken to isolate the symptomatic individual becomes long. Coccia [21] also studied the accelerated transmission dynamics of COVID-19 of Italy, to understand the patterns underpinning the spread of the virus with respect to temporal and spatial domains. His findings showed among others that the accelerated and immense spread of COVID-19 in North Italy was highly associated with air pollution. Rong et al. [22] found that delayed diagnosis and limited hospital resources bring about a burgeoning spread of COVID-19. On the other hand, timely diagnosis and reduction of waiting time has a tendency to cause massive reduction in transmission risk and rising peak levels of new confirmed cases and new infections; which would in the long run will cause a progressive decline in cumulative number of confirmed cases and total infection [23-26].

Governments across Africa with the aim of minimizing further spread of the COVID-19 rolled out stepwise/and or partial non-pharmaceutical control measures such as social distancing, closure of borders, lockdowns, bans on religious and social gatherings and curfews. These measures have culminated into the relatively lower rate of infection being witnessed in Africa compared to western countries. Although, there have been adverse effects of these interventions with regards to socio-economic factors and political stability worldwide, the risk factors that account for the rise in number of cases of COVID-19 in Africa mostly included over-crowded slums and urban communities which suffer poor sanitation, acute water shortages as well as other scarcity of basic social amenities and services [27].

In Ghana, the first confirmation case of COVID-19 pandemic was detected in Accra, the capital on 12 March 2020. The outbreak has then spread through all 16 regions of the country. As at 30 September 2020, Ghana had recorded 46,222 cases with 299 deaths. When the first case was confirmed, the country's health sector initiated a COVID-19 response team for tracking and strengthening early detection of cases for containment and expanded contact tracing system to identify contacts of confirmed cases of COVID-19. Contact tracing is a very important activity in controlling most infectious disease out breaks. It allows for the identification, quarantine, and close monitoring of persons who have been in contact with an infected individual. Delays in identifying contacts meant that, contacts that are not identified and followed to monitor for development of the disease and timely isolation have the potential to spread infection with resultant rapid amplification of the outbreak. One of the biggest challenges in Ghana's fight against covid-19 was lapses in contact identification, listing and tracing [28]. This was as a result of rapidly increasing case numbers which placed a strain on human, financial and logistical resources to effectively trace all contacts. Through the implementation of contact tracing, monitoring of field teams (contact tracers) and data management at district and regional levels, the number of new cases identified increased by 64% from 2,655 to 4,131 samples from over 18,000 contacts within a period of one month [28].

Some attempts are evident in scientific literature on modeling the transmission of COVID-19 for Ghana's situation. Ghanaian Scientists at the Noguchi Memorial Institute for Medical Research in their quest to ascertain if there existed any locally emerging novel COVID-19 strain in Ghana, compared six (6) samples from imported cases to nine (9) cases of local transmission who had no travel history. The findings showed no significant difference in the genetic make-up of the pathogen [29]. Another study sought to evaluate how changes in human behavior (resulting from mitigation and suppression measures proposed by health experts in Ghana) has influenced COVID-19 reported cases. The findings of the study revealed a significant decline in the rate of transmission for 30 days. The findings further suggested a peak of confirmed cases in the range of 64th and 74th days of infection [30]. Fosu et al. [31] also developed a compartmental models to explain the transmissibility of the COVID-19 virus by considering quarantine, lockdown and vaccine interventions using SIR and SEIR models. The studied models predicted that in instances where the basic reproductive number does not exceed one, there is likely an indication that COVID-19 will diminish in the next two to four months in Ghana. Again, Wiah et al. [32] used Next Generation Matrix and the Jacobian Matrix methods among others to formulate a mathematical model for the transmission dynamics of COVID-19.Their findings suggested that control measures put in place were critical to reducing the impact of COVID-19 in Ghana. Otoo et al. [33] also developed forecast models to predict COVID-19 cases in nine (9) countries that included China, Spain, South Korea and Ghana based on the data available at the time (May 10, 2020). The ten (10) days forecast showed that Ghana's cases would not drop any time soon.

In spite of all the interventional measures adopted by most African countries including Ghana, not much work has been carried-out to elucidate country-based transmission patterns of COVID-19 in juxtaposition to the three phases of latency, accelerated and delayed holistically. In this paper, we present a new model for the COVID-19 reported cases and deaths of Ghana in the aforementioned phases.

In the present work, we will expose in a semi-empirical study, the pandemic propagation behavior in exponential form versus time with only three adjustable parameters in order to offer a reliable prediction and estimation, as well as to contribute to the improvement and the advancement of certain theories by proposing a particular solution of their systems of theoretical equations.

2. REPORTED CASES AND DEATHS DATA

2.1 Data Scope

The investigation data of reported cases and deaths of Covid-19 in Ghana are collected from three principal electronic sources [8,9,23] for about eight months (from March 13, 2020 to November 18, 2020). Data are given in Table 1S in the Supplementary Materials Section and are depicted in Fig. 1. In view of adequate empirical expressions, we propose to divide the time range into three domains according to the different trends of curvatures showed in Fig. 1. Table 1 indicates the three main phases, such as (I): latent phase; (II): accelerated phase and (III): the delayed phase.

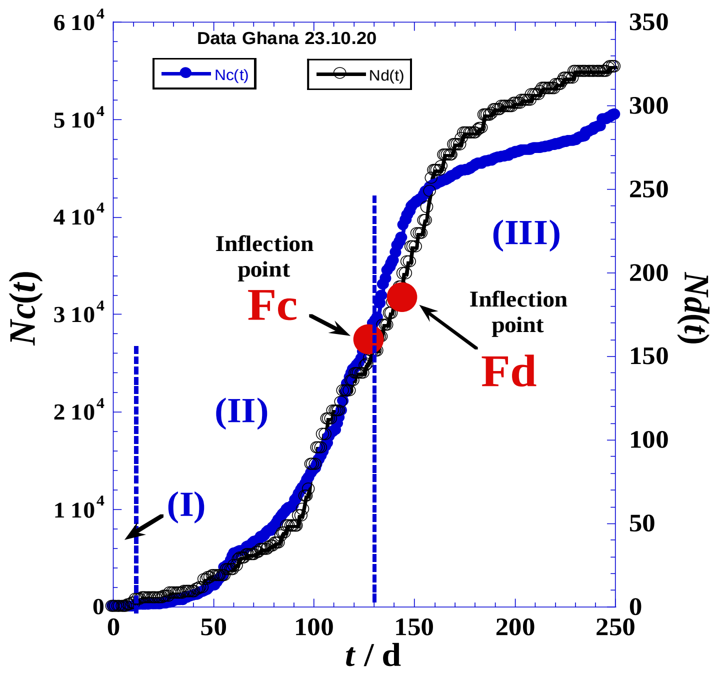

Fig. 1. Total reported cases and deaths for the first 250 days of the pandemic in Ghana

Table 1. Different spread phases and identification of the accelerated phase for the reported cases and deaths

Phase 0		Phase I		Phase II		Phase III
Reported cases						
Absence	$t=t_{c0}$	Latent	$t=t_c$	Accelerated	$t=t_{c1}$	Delayed
Deaths						
Absence	$t=t_{d0}$	Latent	$t=t_d$	Accelerated	$t=t_{d1}$	Delayed

However, R. Baldwin [1] considers that the two phases (0) and (I) in Table 1 constitute the Pre-pandemic intervals and can be assigned as the two stages of Investigation and Recognition, while the phases (II) and (III) constitute the Pandemic intervals and can be assigned as the four stages of Initiation, Acceleration, Deceleration and Preparation.

2.2 Delimitation of Phases' Domains

The accurate determination of the coordinates of inflection points F_c or F_d (Fig. 1 and Fig. 2S) are delimited to the two domains II & III. This is clearly seen from Fig. 1 and Table 1, where Fc (t_{c1}, N_{c1}) and Fd (t_{d1}, N_{d1}) correspond respectively to daily cases and daily deaths. It is noteworthy that (t_{c0}) and (t_{d0}) which are initial times (for cases and deaths) of phase (I) correspond to a day prior to the first non-null apparition of a new case (refer to Table 2 and 1S). Additionally, the initial times (t_{c1}) and (t_{d1}) of the latent phase (I) become the final time for the accelerated phase (II) which are determined through varied methods such as optimization techniques, least square methods and the non-linear regression (see Section 3). Also, derivative and tangent methods were used to determine times (t_{c1}) and (t_{d1}) of the inflection points (F_c) and (F_d) in Fig. 1.

2.2.1 Derivations method

The use of nonlinear regression with low degree polynomial is an effective technique for smoothing minor portions of the curve that show similar curvatures. Subsequently, the derivative of each part (Eq. 1) is found whilst ensuring continuity and differentiability for each boundary. The maximum that occurs at the inflection times (t_{c1}) and (t_{d1}) is reached through the derivative function $N_i(t)$ with respect to the time t.

$$n_i(t) = \frac{dN_i(t)}{dt} \qquad (1)$$

In spite of that; some difficulties may be encountered in modeling; ascribable to irregularities of curvature. In this instance, relative variation for a very small interval of time (Eq. 2) or daily reported cases obtained from the available data can be utilized.

$$n_i(t) = \frac{\Delta N_i(t)}{\Delta t} \qquad (2)$$

Practically, inflection times (t_{c1}) and (t_{d1}) can be approximately determined provided the daily cases reach a smoothed peak maximum (Fig. 2). In point of fact, inflections points (Fc) and (Fd) come about where cases t_{c1} and deaths t_{d1} reach their daily maximum. As a result, we consider their corresponding coordinates as $N_{c1} = N_c$ ($t = t_{c1}$) and $N_{d1} = N_d$ ($t = t_{d1}$) respectively (Table 1).

Observing the Fig. 1, we note that the second peak in Fig. 2b corresponds to the inflection point (F_d) while the first one is due to sudden change of curvature in Fig. 1 giving local and non global inflection point. (The duplicated data declared by the authorities in Table 1S are replaced by intermediate values using interpolation methods through a designed program presented in Supplementary Materials).

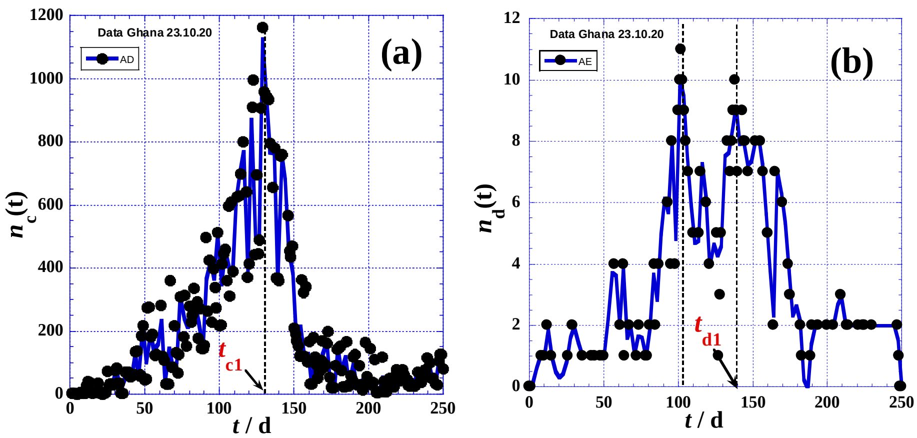

Fig. 2a. new daily reported cases and fig. 2b. new daily reported cases for new daily deaths the first 250 days of the pandemic in Ghana

3. ACCELERATED PHASE MODELING

There are several disturbances in the tendency of the scatter points; the shape of curvature presents certain distortions that they cannot be considered as several independent waves in such a way that it can be assumed that it was globally a single wave but with different amplitudes of fairly close values (Fig. 1). So, we can apply a single model where the adjustable and optimal parameters have average values describing the global accelerated spread of pandemic.

The second constraint for our modeling is that several values of the cumulative cases are given as a couple or triplet of identical consecutive values. This is because the reported cases of Covid-19 are given by the authorities every two or three days (Table 1S) for which the software of the data base includes by default the intermediate values identical to the last declared value. This causes an apparition of some piecewise functions in graphical representations and a lot of zero values in the graphics of new cases (Fig. 1S), which makes the optimization methods less efficient. For this, we kept the last declared value and replaced the other by an intermediate value using the interpolations method and thanks to an elaborate program (Supplementary Materials).

Having determined the two inflection times (t_{c1}) and (t_{d1}) representing the second boundary of the accelerated phase (II), and through nonlinear regression by optimizing the standard deviation (σ) and the relative error (*Erel*), the first limit of times (t_c) and (t_d) can be sufficiently known. To this end, the relative error (*Erel*) between experimental and estimated values (Tables 1S) as determined by the proposed model are determined. Conversely, as a result of the pseudo-Gaussian shape of the derivative function $dN(t)/d(t)$ which are plotted in Fig. 2, the following three independent adjustable parameters for the reported cumulative cases $Nc(t)$ and the cumulative deaths $Nd(t)$ in the accelerated phase (II) that have been recorded have been suggested;

$$N_c(t) = N_{c0}\left(e^{\frac{(t-t_c)}{\tau_c}} - 1\right) + \delta N_c \quad (3)$$

$$N_d(t) = N_{d0}\left(e^{\frac{(t-t_d)}{\tau_d}} - 1\right) + \delta N_d \quad (4)$$

Where increments of (δNc) and (δNd) are reliant on parameters and can be adjusted to the values of the reported cases $Nc(t_c)$ and the deaths $Nd(t_d)$ at the start of the accelerated phase (II), with optimization being the main preferred means irrespective of if there is a slight difference from the experimental values shown in Table 1S.

$$\delta N_c \approx N_c(t_c) \quad (5)$$

And

$$\delta N_d \approx N_d(t_d) \quad (6)$$

The optimal values reported for the adjustable parameters for the accelerated phase (II) cases and deaths as determined by non-linear regressions for the Eqs. 3 and 4 are identifiable from Table 2.

We note that (A_{c0}) and (A_{d0}) denote the cases' activity and the deaths' activity expressed as follows:

$$A_{c0} = \frac{N_{c0}}{\tau_c} \quad (7)$$

And

$$A_{d0} = \frac{N_{d0}}{\tau_d} \quad (8)$$

Fig. 3 shows an acceptable agreement between the experimental values and the estimated ones in the accelerated phase (II), while the observed discrepancy in the delayed phase (III) for which it leads us to slightly modify the model of Eq. 3 while keeping the same general exponential form to predict the slower phase (II) in the following Sections 5 and 6.

Table 2. Different Spread Phases and Identification of the Accelerated Phases

t_{c0}	t_c	t_{c1}	τ_c	N_{c0}	*Erel*	σ	N_{c1}	A_{c0}
0	27	129	47.25	3675	4.81%	508.4	28430	77.78
t_{d0}	t_d	t_{d1}	τ_d	N_{d0}	*Erel*	σ	N_{d1}	A_{d0}
7	42	138	37.262	19.5	17.98%	11.96	175	0.5233

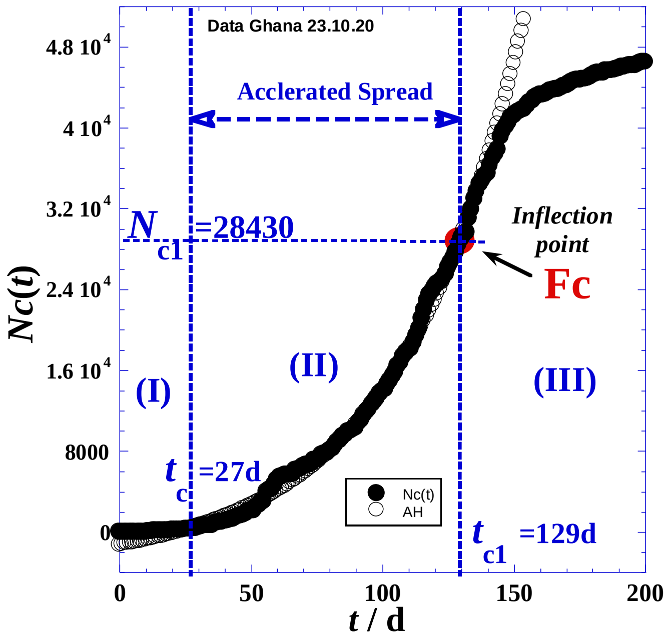

Fig. 3a. The total reported cases *Nc*(*t*) for the first 200 days of the pandemic in Ghana. For the accelerated phase (II) using Eq. 3

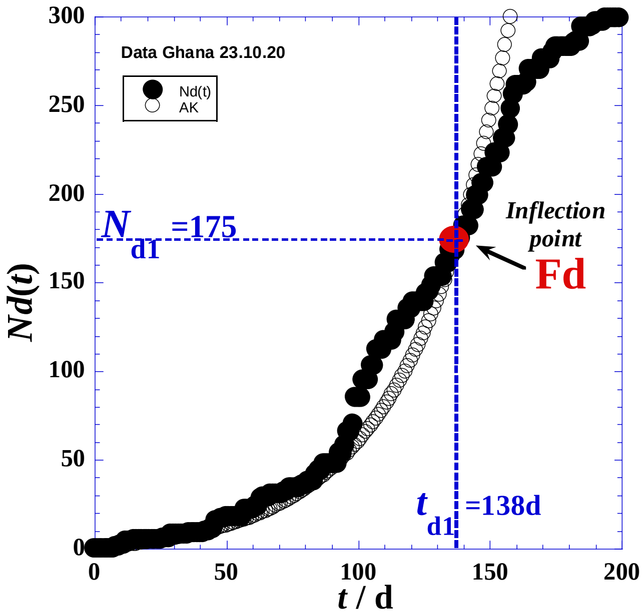

Fig 3b. The Total Reported Deaths *Nd*(*T*) for the First 200 Days of the Pandemic in Ghana; for the Accelerated Phase (II) using Eq. 4

According to the work by [34,35,36], the transmission of COVID-19 will not depict exponential curve and if it does, it will be due to errors since it is an epidemiological curve (not a theory or hypothesis) where number of new cases increases rapidly, peaks, and then drops.

The findings from the work showed that the spread of COVID-19 rather follow exponential function. This is attributed to several causes for the complicated propagation, which is best depicted exponentially. We propose an empirical model, where we begin our modeling with the form (Eq. 3) based on Figure 4S which justifies our choice, where the linearity of the logarithm can be realized in a wide range of time in the accelerated phase (II), which is the object of the proposed model.

Again, the peak height (Fig. 2a, Eq. 9) by virtue of the maximum derivative function of *Nc*(t_{c1}) which occur at the highest day (t_{c1}) coupled with inflexion point (Fc) for the *Nc*(*t*)-curve (Fig. 1) which is an indicating of very weak containment policies and negligible interventions [1,31]. The curve assumes diverse shapes, depending on the infection rate of the virus and the health system capacity [12].

$$n_{c,max}(t = t_{c1}) = A_{c0}e^{\frac{(t_{c1}-t_c)}{\tau_c}} \quad (9)$$

4. REPORTED CASES-DEATHS CORRELATION

Considering the present work as an empirical investigation, this section will just introduce empirical comparisons in order to help the theoreticians to invest in more details in their investigations of the handled theoretical parameters.

As a first examination from Table 2, we can write the following inequality equations:

$$\begin{cases} t_d > t_c \\ N_{d0} < N_{c0} \\ \tau_d < \tau_c \end{cases} \quad (10)$$

which it must be taken as mathematical necessary conditions and main constraints in optimization problems for Ghana specificity. We can also add the following derived parameters necessary for subsequent discussions and interpretations:

$$\begin{cases} \Delta t = t_d - t_{dc} \\ \Delta N = N_{c0} - N_{d0} \\ \Delta\tau = \tau_d - \tau_c \end{cases} \quad (11)$$

$$A_{d0} < A_{c0} \quad 12)$$

One of the ways to see the mutual correlation between the reported cumulative cases *Nc*(*t*) and the cumulative deaths $N_d(t)$ is to eliminate the time-variable and plot $N_d(t)$ as a function of *Nc*(*t*) in Fig. 4. We observe an interesting linear dependence in a domain stretched between the two accelerated (II) and delayed (III) phases. After that, the positive deviation to the linearity (with high slope value) indicates that each reported cases phase always precedes in time the similar phase related to deaths cases.

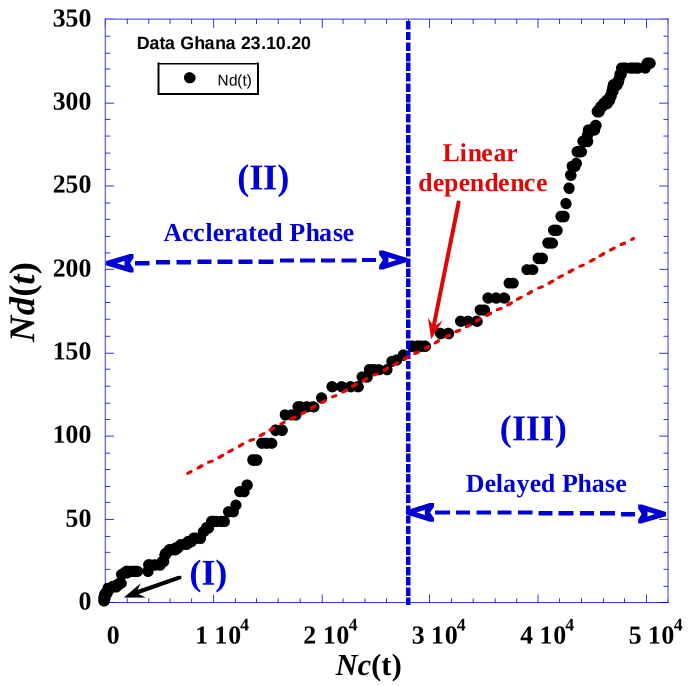

Fig. 4. Cumulative Deaths $N_d(t)$ versus the total reported cases *Nc*(*t*)for the first 250 days of the Pandemic in Ghana

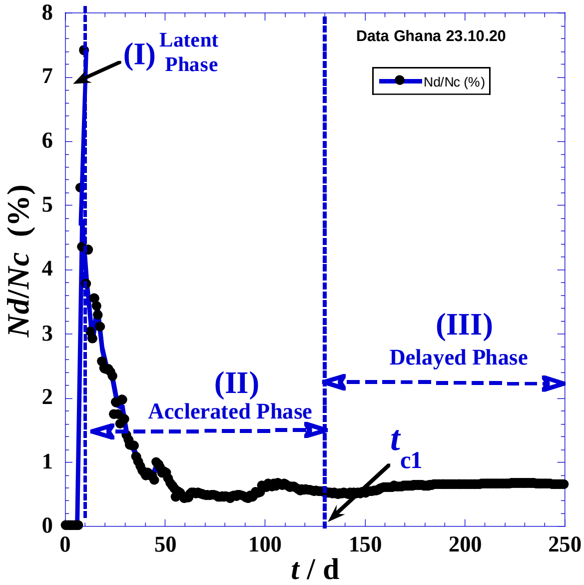

Fig. 5. The mortality rate *T*(*T*) as function of the time for the first 250 days of the pandemic In Ghana

Another manner of comparison consists to introduce the mortality rate $T(t)$ expressed as follows:

$$T(t) = \frac{N_d(t)}{N_c(t)} \quad (13)$$

The graphical representation of the mortality rate $T(t)$ in percent is depicted in Fig. 5. We can observe distinct behaviors of the three spread phases. The maximum occurring very earlier and before the accelerated phase at ($t = t_{dc}$) about 10 days, is mathematically due to (*i*): the strong correlation cases-deaths, and (*ii*): the sign conflict between the two logarithms $\ln N_d(t)$ and $\ln Nc(t)$ which is clearly revealed in Fig. 5S. We can conclude that we can benefit from this feature by following this variation over time since the beginning of the spread, and when the mortality rate $T(t)$ reaches the maximum, we can predict that the pandemic is preparing to move from the accelerated phase I to the delayed phase II if there are no great changes in the precautionary measures and the peoples' behaviors towards the Covid-19 pandemic.

5. PREDICTION OF DELAYED PHASE FOR SYMMETRIC BEHAVIOR

To predict the delayed phase (III) from only the accelerated phase (II) data, we must consider as a first approximation, that the kinetic progress of the Covid-19 pandemic is the same before and after the highest day (t=t_{c1}). This symmetric behavior occurs when there are no changes in the environments of the pandemic, such as the precautionary measures and the peoples 'behaviors towards the Covid-19 pandemic, etc. On the other hand, the symmetric behavior is translated mathematically by the fact that the inflection point (Fc) will be a center of symmetry of the curve in Fig. 1.

So, when we respect the boundary conditions, continuity and the derivability at the inflection point Fc(t= t_{c1}), we can obtain the equation predicting the delayed phase (III) expressed as follows:

$$\mathrm{N_c(t)} = \mathrm{N_{c1}}\left(2 - \mathrm{e}^{\frac{-(\mathrm{t}-\mathrm{t_{c1}})}{\tau'_\mathrm{c}}}\right) \quad (14)$$

$$\tau'_\mathrm{c} = \frac{\tau_\mathrm{c}\mathrm{N_{c1}}}{\mathrm{N_{c0}}}\mathrm{e}^{\frac{-(\mathrm{t_{c1}}-\mathrm{t_c})}{\tau_\mathrm{c}}} \quad (15)$$

In our case, (τ'_c = 46 days). We note that the precedent values are close to (τ_c) given in Table 2 for the accelerated phase (II). Therefore, we can conclude that in a reliable approximation, we can simplify the problem and put the value of (τ_c) in Eq. 14 in place of (τ'_c) without any net imprecision (Fig. 6). The discrepancy between experimental values and estimated ones within 170 days is due to that the process of spread is not symmetric.

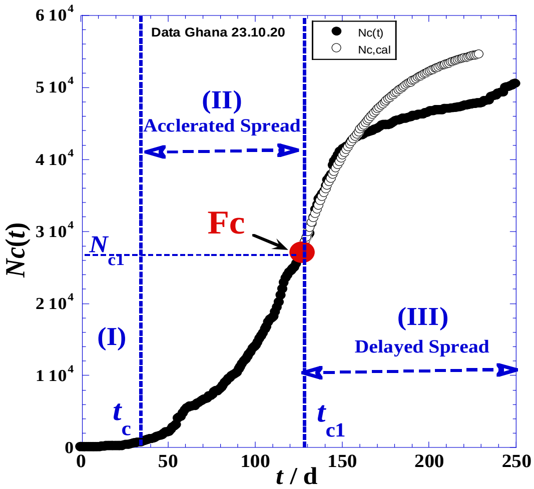

Fig. 6. Total reported cases *Nc*(*t*) for the first 250 days for delayed phase (III) in symmetric behavior using Eq. 14 and τ_c'= 46 days

6. PREDICTION OF DELAYED PHASE FOR ASYMMETRIC BEHAVIOR

Generally, and for real situations, we cannot observe the symmetric behavior previously mentioned because of the instantaneous change of people behaviors and authority decisions. So, respecting the continuity and the derivability on the highest day (t= t_{c1}) occurring at the inflection point Fc (t= t_{c1}), the equation predicting the delayed phase (III) becomes expressed as follows:

$$N_c(t) = N_{c1} + \frac{\tau'_c N_{c0}}{\tau_c}\left(1 - e^{\frac{-(t-t_{c1})}{\tau'_c}}\right) e^{\frac{(t_{c1}-t_c)}{\tau_c}} \quad (16)$$

Here, only one adjustable parameter (τ'_c) is needed to be estimated using optimizations techniques. The downside of this situation is that we can't apply any non-linear regression if we don't have enough data points after the highest day (t= t_{c1}). However, a successful prediction should be also in agreement with the limiting value (Nc_∞) of the reported cumulative case at the end of the Covid-19 pandemic (Eq. 17).

$$N_{c\infty} = N_{c1} + \frac{\tau'_c N_{c0}}{\tau_c} e^{\frac{(t_{c1}-t_c)}{\tau_c}} \quad (17)$$

Fig. 7 shows net improvement relative to the symmetric prediction using in Eq. 17 an optimal value (τ'_c = 24.5 days) determined by the least square method of non-linear regression.

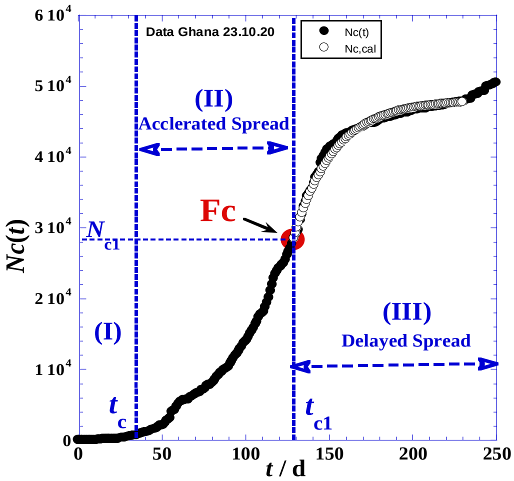

Fig. 7. Total reported cases *Nc*(*t*) of the first 250 days for delayed phase (III) in asymmetric behavior using Eq. 16 with τ_c'= 24.5 days and $N_{c\infty}$ = 48000

We can see that starting from 230 days, there is an apparition of a new wave with high amplitude than the previous one.

7. DISCUSSION

The findings of the research are contrary to the findings of Gu et. al [37], that COVID-19 cases follow cubic and quadratic distributions respectively for China and Hubei. This suggests that there is a possible disparity in the distribution trend that country-based COVID-19 cases must follow worldwide. A modification of the developed model to help optimize the error between observed and estimated values yielded an improvement in the prediction of the delayed phase. The transmission of the virus between phases depended on changes in the precautionary measures and peoples' behaviors.

8. CONCLUSION

From the study, a new accelerated spread and prediction of the delayed phase model for the COVID-19 reported cases and deaths of Ghana has been developed. The developed model shows that the distribution of the COVID-19 situation in Ghana followed an exponential function. The study also revealed that Ghana has passed her inflection point and is currently in the delayed phase characterized with a staggering trend where new infections are similar in magnitude to previous infections over a long period of time. Ghana reaching her tipping point is the manifestation of successful adherence to public-health intervention or preventive measures. In summary, a commencement of a new wave in Ghana with higher intensity than what was previously observed was predicted. This was shown by an incremental decrease for consecutive days in the trend.

9. RECOMMENDATION

Anti-transmission COVID-19 policies need to be continuously enforced in Ghana to prevent the predicted second wave of the virus spread.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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APPENDICES

1S. Data Collection

We have collected data from electronic sources [A,B,C], for about 240 days for Ghana (from March 13, 2020 to November 18, 2020).

[A] COVID-19 dashboard, Center for Systems Science and Engineering (CSSE), Johns Hopkins University (JHU) 2020. https://systems.jhu.edu/
[B] COVID-19 Coronavirus Pandemic, https://www.worldometers.info/coronavirus/
[C] Github, Our World in Data, https://github.com/owid/covid-19-data/tree/master/public/data

Table 1S. Data of Reported cases and deaths of Covid-19 for Ghana for about eight months (from March 13, 2020 to November 18, 2020)

Date	Total cases	Total deaths	Time (t) / day	Date	Total cases	Total deaths	Time (t) / day
3/13/2020	0	0	0	08/11/2020	41404	215	151
3/14/2020	3	0	1	08/12/2020	41572	223	152
3/15/2020	6	0	2	8/13/2020	41725	223	153
3/16/2020	6	0	3	8/14/2020	41847	223	154
3/17/2020	7	0	4	8/15/2020	42210	231	155
3/18/2020	7	0	5	8/16/2020	42532	231	156
3/19/2020	11	0	6	8/17/2020	42653	239	157
3/20/2020	16	0	7	8/18/2020	42993	248	158
3/21/2020	19	1	8	8/19/2020	43094	256	159
3/22/2020	23	1	9	8/20/2020	43260	261	160
3/23/2020	27	2	10	8/21/2020	43325	261	161
3/24/2020	53	2	11	8/22/2020	43325	261	162
3/25/2020	93	4	12	8/23/2020	43505	261	163
3/26/2020	132	4	13	8/24/2020	43622	263	164
3/27/2020	137	4	14	8/25/2020	43717	270	165
3/28/2020	141	5	15	8/26/2020	43769	270	166
3/29/2020	152	5	16	8/27/2020	43841	270	167
3/30/2020	152	5	17	8/28/2020	43949	270	168
3/31/2020	161	5	18	8/29/2020	44118	270	169
4/01/2020	195	5	19	8/30/2020	44205	276	170
4/02/2020	204	5	20	8/31/2020	44298	276	171
4/03/2020	205	5	21	09/01/2020	44460	276	172
4/04/2020	205	5	22	09/02/2020	44658	276	173
4/05/2020	214	5	23	09/03/2020	44713	280	174
4/06/2020	214	5	24	09/04/2020	44777	283	175
4/07/2020	287	5	25	09/05/2020	44777	283	176
4/08/2020	313	6	26	09/06/2020	44777	283	177
4/09/2020	378	6	27	09/07/2020	44869	283	178
4/10/2020	378	6	28	09/08/2020	45012	283	179
4/11/2020	408	8	29	09/09/2020	45313	283	180
4/12/2020	566	8	30	09/10/2020	45313	283	181
4/13/2020	566	8	31	09/11/2020	45388	285	182
4/14/2020	636	8	32	09/12/2020	45434	286	183
4/15/2020	636	8	33	9/13/2020	45434	286	184
4/16/2020	641	8	34	9/14/2020	45601	294	185
4/17/2020	641	8	35	9/15/2020	45655	294	186
4/18/2020	834	9	36	9/16/2020	45655	294	187
4/19/2020	1042	9	37	9/17/2020	45714	294	188
4/20/2020	1042	9	38	9/18/2020	45760	295	189

Date	Total cases	Total deaths	Time (t) / day	Date	Total cases	Total deaths	Time (t) / day
4/21/2020	1042	9	39	9/19/2020	45877	297	190
4/22/2020	1154	9	40	9/20/2020	46004	297	191
4/23/2020	1154	9	41	9/21/2020	46062	297	192
4/24/2020	1279	10	42	9/22/2020	46062	297	193
4/25/2020	1279	10	43	9/23/2020	46153	299	194
4/26/2020	1550	11	44	9/24/2020	46222	299	195
4/27/2020	1550	11	45	9/25/2020	46222	299	196
4/28/2020	1671	16	46	9/26/2020	46222	299	197
4/29/2020	1671	16	47	9/27/2020	46387	299	198
4/30/2020	2074	17	48	9/28/2020	46444	299	199
5/01/2020	2074	17	49	9/29/2020	46482	301	200
5/02/2020	2169	18	50	9/30/2020	46626	301	201
5/03/2020	2169	18	51	10/01/2020	46656	301	202
5/04/2020	2719	18	52	10/02/2020	46694	301	203
5/05/2020	2719	18	53	10/03/2020	46803	303	204
5/06/2020	3091	18	54	10/04/2020	46829	303	205
5/07/2020	3091	18	55	10/05/2020	46829	303	206
5/08/2020	4012	18	56	10/06/2020	46829	303	207
5/09/2020	4263	22	57	10/07/2020	46829	303	208
5/10/2020	4263	22	58	10/08/2020	46947	306	209
5/11/2020	4700	22	59	10/09/2020	46987	306	210
5/12/2020	5127	22	60	10/10/2020	47005	306	211
5/13/2020	5408	24	61	10/11/2020	47005	306	212
5/14/2020	5530	24	62	10/12/2020	47030	308	213
5/15/2020	5638	28	63	10/13/2020	47126	310	214
5/16/2020	5735	29	64	10/14/2020	47126	310	215
5/17/2020	5735	29	65	10/15/2020	47173	310	216
5/18/2020	5735	29	66	10/16/2020	47173	310	217
5/19/2020	6096	31	67	10/17/2020	47232	310	218
5/20/2020	6269	31	68	10/18/2020	47310	310	219
5/21/2020	6269	31	69	10/19/2020	47372	310	220
5/22/2020	6486	31	70	10/20/2020	47461	312	221
5/23/2020	6617	31	71	10/21/2020	47461	312	222
5/24/2020	6683	32	72	10/22/2020	47538	312	223
5/25/2020	6808	32	73	10/23/2020	47601	314	224
5/26/2020	7117	34	74	10/24/2020	47690	316	225
5/27/2020	7303	34	75	10/25/2020	47690	316	226
5/28/2020	7303	34	76	10/26/2020	47775	316	227
5/29/2020	7616	34	77	10/27/2020	47775	316	228
5/30/2020	7768	35	78	10/28/2020	47775	316	229
5/31/2020	8070	36	79	10/29/2020	48055	320	230
06/01/2020	8070	36	80	10/30/2020	48055	320	231
06/02/2020	8297	38	81	10/31/2020	48055	320	231
06/03/2020	8548	38	82	11/01/2020	48124	320	233
06/04/2020	8885	38	83	11/02/2020	48200	320	234
06/05/2020	9168	42	84	11/03/2020	48200	320	235
06/06/2020	9462	44	85	11/04/2020	48643	320	236
06/07/2020	9638	44	86	11/05/2020	48788	320	237
06/08/2020	9910	48	87	11/06/2020	48788	320	238
06/09/2020	10201	48	88	11/07/2020	48904	320	239
06/10/2020	10201	48	89	11/08/2020	49102	320	240
06/11/2020	10358	48	90	11/09/2020	49202	320	241
06/12/2020	10856	48	91	11/10/2020	49302	320	242
6/13/2020	11118	48	92	11/11/2020	49302	320	243
6/14/2020	11964	54	93	11/12/2020	49957	320	244

Date	Total cases	Total deaths	Time (t) / day	Date	Total cases	Total deaths	Time (t) / day
6/15/2020	11964	54	94	11/13/2020	50018	320	245
6/16/2020	12193	58	95	11/14/2020	50018	320	246
6/17/2020	12590	66	96	11/15/2020	50123	322	247
6/18/2020	12929	66	97	11/16/2020	50376	323	248
6/19/2020	13203	70	98	11/17/2020	50376	323	249
6/20/2020	13717	85	99	11/18/2020	50457	323	250
6/21/2020	14154	85	100	11/19/2020*	50631*	323*	251*
6/22/2020	14154	85	101	11/20/2020	50631	323	252
6/23/2020	14568	95	102	11/21/2020	50717	323	253
6/24/2020	15013	95	103	11/22/2020	50874	323	254
6/25/2020	15473	95	104	11/23/2020	50941	323	255
6/26/2020	15834	103	105	11/24/2020	51184	323	256
6/27/2020	16431	103	106	11/25/2020	51225	323	257
6/28/2020	16742	112	107	11/26/2020	51225	323	258
6/29/2020	17351	112	108	11/27/2020	51225	323	259
6/30/2020	17741	112	109	11/28/2020	51379	323	260
07/01/2020	18134	117	110	11/29/2020	51569	323	261
07/02/2020	18134	117	111	11/30/2020	51667	323	262
07/03/2020	19388	117	112	12/01/2020	51667	323	263
07/04/2020	19388	117	113	12/02/2020	51667	323	264
07/05/2020	20085	122	114	12/03/2020	51667	323	265
07/06/2020	21077	129	115	12/04/2020	52096	325	266
07/07/2020	21968	129	116	12/05/2020	52096	325	267
07/08/2020	22822	129	117	12/06/2020	52274	325	268
07/09/2020	23463	129	118	12/07/2020	52274	325	269
07/10/2020	23834	135	119	12/08/2020	52500	326	270
07/11/2020	24248	135	120	12/09/2020	52622	326	271
07/12/2020	24518	139	121	12/10/2020	52738	326	272
7/13/2020	24988	139	122	12/11/2020	52738	326	273
7/14/2020	24988	139	123	12/12/2020	52933	327	274
7/15/2020	25430	139	124	12/13/2020	52933	327	275
7/16/2020	26125	139	125	12/14/2020	53014	327	276
7/17/2020	26572	144	126	12/15/2020	53270	327	277
7/18/2020	27060	145	127	12/16/2020	53386	327	278
7/19/2020	27667	148	128	12/17/2020	53553	331	279
7/20/2020	28430	153	129	12/18/2020	53653	331	280
7/21/2020	28989	153	130	12/19/2020	53653	331	281
7/22/2020	29672	153	131	12/20/2020	53653	331	282
7/23/2020	29672	153	132	12/21/2020	53954	333	283
7/24/2020	31057	161	133	12/22/2020	53954	333	284
7/25/2020	31851	161	134	12/23/2020	54043	333	285
7/26/2020	32969	168	135	12/24/2020	54043	333	286
7/27/2020	33624	168	136	12/25/2020	54043	333	287
7/28/2020	34406	168	137	12/26/2020	54286	333	288
7/29/2020	35142	175	138	12/27/2020	54401	333	289
7/30/2020	35142	175	139	12/28/2020	54503	333	290
7/31/2020	35501	182	140	12/29/2020			
08/01/2020	37014	182	141	12/30/2020			
08/02/2020	37014	182	142	12/31/2020			
08/03/2020	37812	191	143	01/01/2021			
08/04/2020	37812	191	144	01/02/2021			
08/05/2020	39075	199	145	01/03/2021			
08/06/2020	39642	199	146	01/04/2021			
08/07/2020	40097	206	147	01/05/2021			
08/08/2020	40533	206	148	01/06/2021			

Date	Total cases	Total deaths	Time (t) / day	Date	Total cases	Total deaths	Time (t) / day
08/09/2020	41003	215	149	01/07/2021			
08/10/2020	41212	215	150	01/08/2021			

** Additional data collected after this investigation[C], we hope it will be useful for those who intend to do further work*

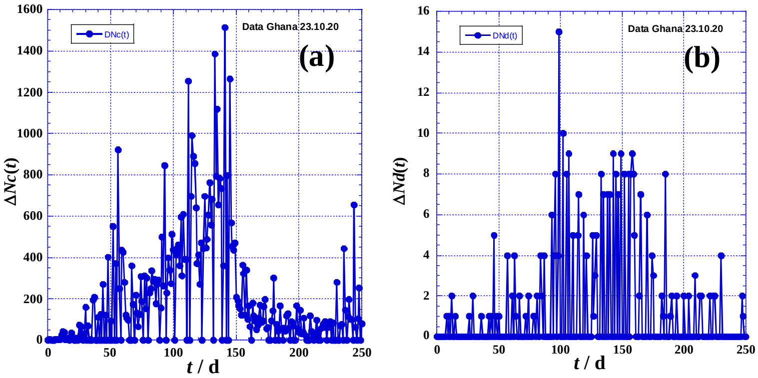
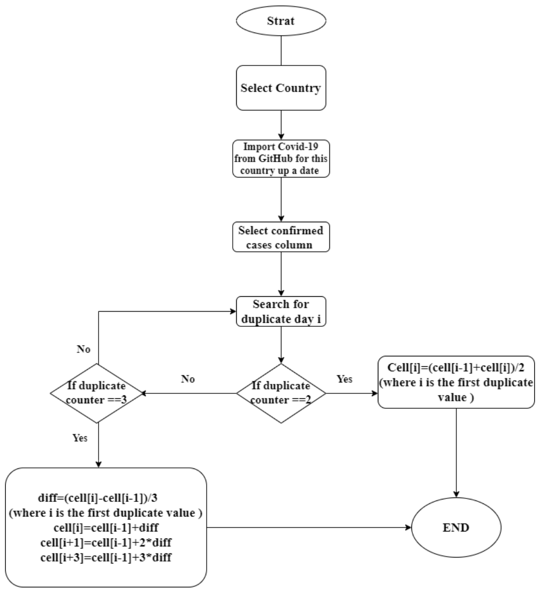


Fig. 1S. New daily reported cases (a) and new daily deaths (b) for the first 250 days of the pandemic in Ghana (Reported from original data without smoothing of duplicated values)

2S. Elimination of Duplicated Data

There is a difference in how countries record covid-19 data because there is no accepted international standard for how they measure this information.
This reflects the presence of duplicate information in the excel sheet that is why we have proposed the flowchart for adjustment this duplicate data.

- We suppose i is the first duplicate data always
- (i-1)the day before the duplicate data
- (i+1) is the day after the first duplicate data.



Scheme 1S. Flowchart for Adjustment the Duplicate Data in Excel Sheet

3S. Determining Intervals of Concavity and Inflection Points

The tangent method involves the use of graphical technique in determining the inflection points. The method begins with drawing three lines: The first two referred to as the parallel tangents which are tangential to the curve on either side of the jump of *Nc* (or *Nd*), the third straight line is parallel to aforementioned tangents and equidistant to each of them.The third straight line will tend to intersect the curve at the midpoint of the jump of *Nc* (or *Nd*), where the coordinates of this intersection point (Figs. 2S and 3S) will correspond to the coordinates of the inflection point Fc(t_{c1},Nc_1) or Fd(t_{d1},Nd_1).

It is noticeable that an inflection point is a point where there is a change in concavity (Fig. 1). The method of determining the intervals of concavity is same as the method used to determine the intervals of increase/decrease as shown in Fig. 2, except that the second derivative is used instead of the first derivative (Fig. 3S). Specifically, as$d(dN/dt)/dt=d^2N/dt^2$, the intervals of increase/decrease for the first derivative in Fig. 2 will determine the concavity of $N(t)$. The two extrema of Fig. 3S indicate where the two tangent lines are positioned as optimal positions.

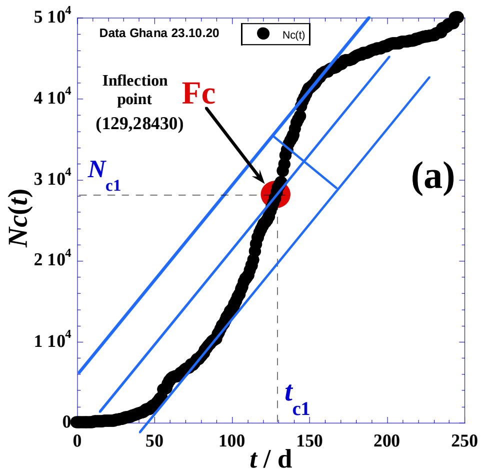


Fig. 2Sa.Graphical Determination of the Inflection point Fc(t_{c1},N_{c1}) Related the Total Reported Cases $Nc(t)$ for the first 250 days in Ghana

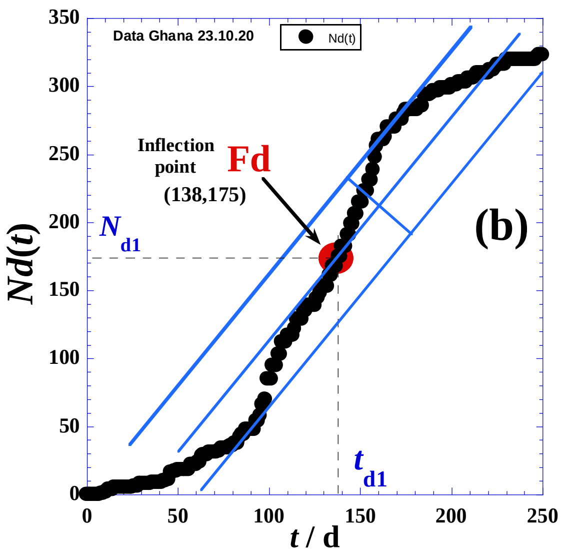


Fig. 2Sb. Graphical Determination of the Inflection point Fd(t_{d1},N_{d1}) Related the Total Death Cases $Nc(t)$ for the first 250 days in Ghana

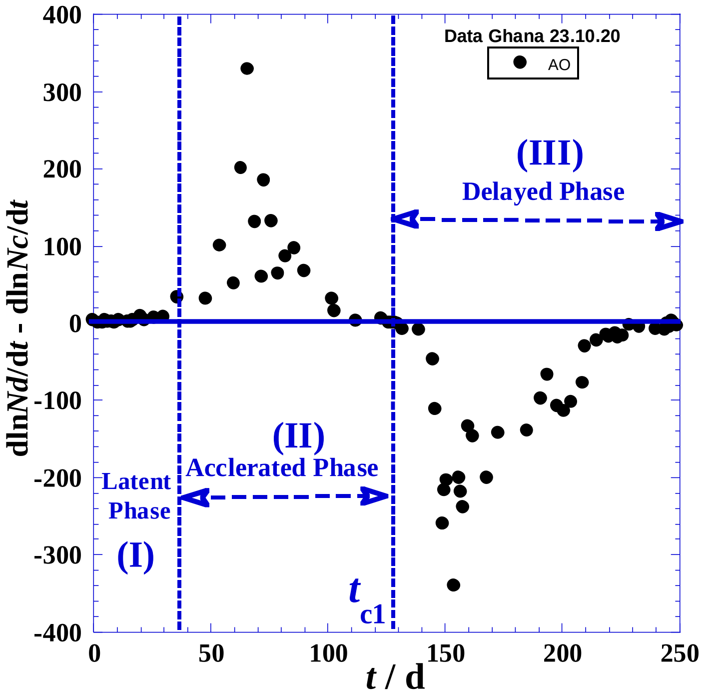


Fig. 3S. Second Derivative Function (d^2Nc/dt^2) Related the Total Death Cases *Nc*(*t*) for the first 250 days in Ghana

Here, we have avoided the aberrant values (with opposite sign) due to fluctuations and double derivative operations.

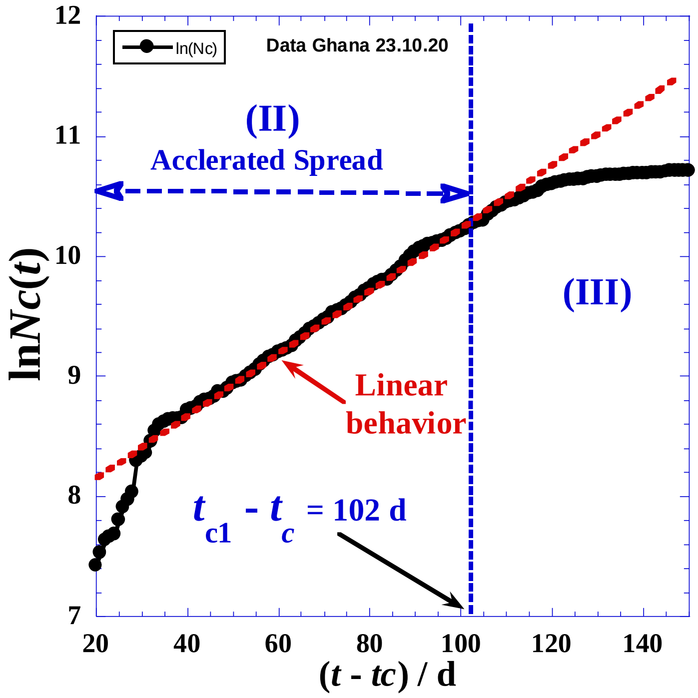


Fig. 4S.Natural Logarithm of Total Reported Cases ln*nc*(*T*) With Time In The Accelerated Phase (II)

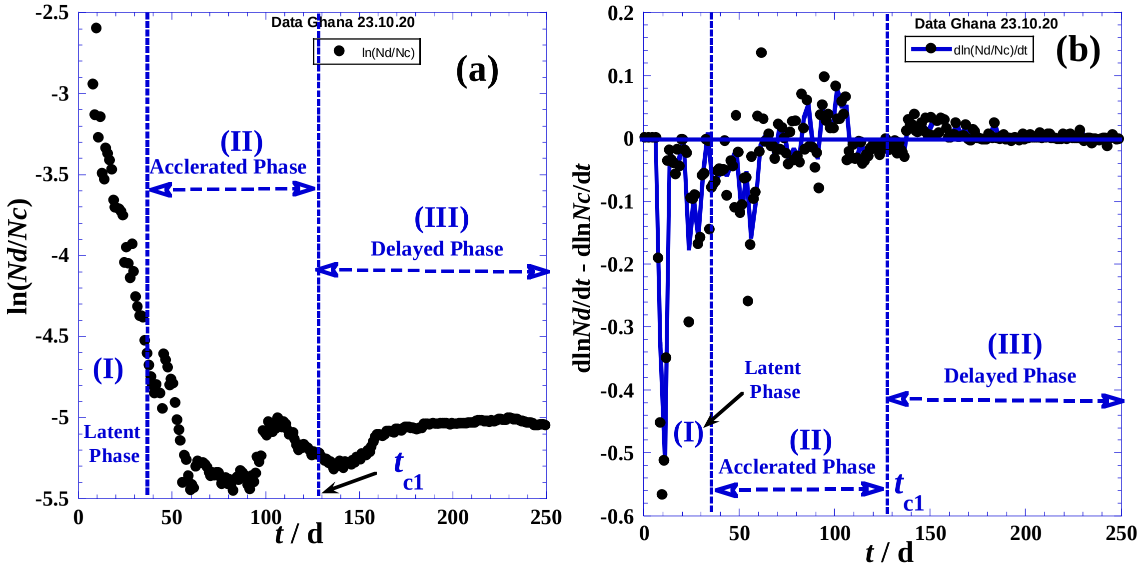


Fig. 5S. Difference between logarithm of the derivative functions of the total reported cases and deaths; to confirm the global maximum occurring ln*Nc*/*Nd*-Curve (Fig. 5)

Due to perturbations in data of scatter points, we cannot see clearlywhere this function cuts the time axis. Nevertheless, we are sure that on approximately the time ($t = t_{c1}$) must cuts the zero for the highly day illustrated by the inflection point in Fig. 1 and a maximum in Fig. 2. In the same context, we can predict the final value of the ratio Nd/Nc of Fig. 5 in the case of symmetric behavior by the following expression:

$$\lim_{\infty} \frac{N_d(t)}{N_c(t)} = \frac{N_{d0}}{N_{c0}} e^{\frac{t_c}{\tau_c} - \frac{t_d}{\tau_d}} \quad (1S)$$

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