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Growth Models for Covid-19 Death Figures of Turkey

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Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

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ABSTRACT

This paper presents modeling of the COVID-19 pandemic deaths to understand behavior of it, predict the peak point of the deaths and cases and produces a short-term forecast using the growth models for the reported data of Turkey. The data which is used in this study are gathered of daily announced by Minister of Health. Von Bertalanffy model has outperformed to the other models considered in this study. However, exponential model has predicted the total deaths and total cases better than the others. And, exponential model has given the best prediction errors among them regarding to the death and positive case figures for last months. Observed data have tended to increase since the last days of August. This could mean that the COVİD-19 threat has reached to a critical stage to crack down on prevention of pandemics spread. Or it could sign the beginning of a second wave of epidemics. More studies must be realized to learn more about the pandemic when the new data are available.

Keywords: Pandemic; forecasting of COVID-19; growth models; positive case and death figures of Turkey.

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1. INTRODUCTION

The new corona virus (SARS-CoV-2) spread widely in China, and a large number of people became infected from the outbreak in December 2019. The World Health Organization (WHO) declared the outbreak is a public health emergency of international concern on 30 January 2020 [1]. On 11 February, WHO announced a name for the new coronavirus disease as the COVID-19 [2], and on 11 March, the COVID 19 outbreak as pandemic [3]. The new corona virus has a great threat to the health and safety of people all over the world due to its amazing spreading power and potential harms [4, 5]. As of 31 August 2020, more than 23.9 million cases have been reported across 188 countries and territories, resulting in more than 819.000 deaths [6]. Turkey is one of these countries and brief history as follows.

The COVID-19 pandemic is still ongoing process in Turkey as of August. The disease was first confirmed to have reached Turkey on 11 March 2020. The first death due to the COVID-19 in the country occurred on 15 March and, it was declared that COVID-19 had spread all over Turkey on 1 April. On 14 April, it is announced that the spread of the virus in Turkey has reached its peak in the fourth week and started to slow down. On 6 May, it was spoken about the normalization process in Turkey. The normalization process in Turkey have been started step by step from 1 June. During the COVID-19 pandemic process all the daily data related the disease have been announced by the Ministry of Health. [7]. As of 31 of August the cumulative deceased patients have been reached to 6370.

The COVID-19 pandemic is the greatest global health crisis on our century we have faced for a long time. The pandemic has also created an unprecedented socio-economic crisis beside a health crisis. It has the potential to create devastating social, economic and political effects that will leave deep and long-standing scars on every one of the countries it spreads [8].

Faced with this unprecedented situation, governments all around the World have been focusing on taking the pandemic under control and reviving their economies again after the lockdown period [9,10, 11].

The crucial question which the researchers (actually all people) have been seeking the answer since the beginning of the pandemic that is how many people will be possibly infected and die of COVID-19 until the end of the pandemic. We still have limited knowledge about the epidemic for example whether the virus has a seasonal effect and a second wave will come. A way to answer these questions is interpretation the behaviors of the pandemic through modeling of the COVID-19 data. There are numerous mathematical models and methods in the literature. We should keep in mind that these models aren't a magical wand to find out everything's about the pandemic. Another important point to be taken into account that lack of correct data. This is caused by misreporting of the cases for many reasons. The studies are so important for decision makers and government leaders to allocate resources to health care facilities, and to take precautions against spreading of the virus.

Numerous mathematical models are being produced to forecast the future of coronavirus disease 2019 (COVID-19) epidemics in the US and worldwide [12]. And there are several academic studies in the literature to model and forecast COVID-19 cases and deaths, and some of them as follows. Bertozzi et al. [13] used exponential growth, self-exciting branching process and susceptible infected resistant (SIR) component models to model cumulative infection cases for South Corea, Italy, Germany, France, Spain, UK, US and Japan. Jia et al. [14] used logistic, Von Bertanffy and Gompertz growth models to model and forecast of cumulative cases and cumulative deaths for China. Perc et al. [15] used a simple iteration-based method to forecast cumulative case for US, Slovenia, Iran and Germany. Anastassopoulou et al. [16] used the method on the basis of a susceptible infectious-recovered-dead (SIRD) model to forecast infected cases for Hubei, China. Binti Hamzah et al. [17] utilize susceptible-exposed-infectious-recovered (SEIR) predictive modelling to forecast the outbreak within and outside of China based on daily observations. Chen et al. [18] used logistic growth model to forecast of cumulative cases for US. Roosa et al. [19] used logistic and Richards growth models for Short-term Forecasts of the COVID-19 Epidemic in Guangdong and Zhejiang, China. Wu et al. [20] employed logistic growth model and the Richards model to analyze the reported number of infected cases in the COVID-19 epidemics for China, Japan, South Korea, Iran, and Italy. Papastefanopoulos et al. [21] used six different time series approaches, namely ARIMA, the

Holt–Winters additive model, TBAT, Prophet, DeepAR and N-Beats to estimate the percentage of active cases with respect to the total population and then compared their performances for USA, UK, Italy, Spain, Russia, France, Turkey, Germany, Iran and Brazil. Salgotra et al. [22] developed genetic programming-based prediction models to forecast confirmed cases and deaths for Maharashtra, Gujarat and Delhi states and whole India.

The purposes of this study are to analyze the behavior of the COVID-19 pandemic figures of Turkey over time and to make short term forecasting of daily deaths and cases. I have used logistic growth, Von Bertanffy growth, exponential growth, Gaussian growth and Richards growth models for modelling stage because of their explanatory and simple form. First, the series between first death day and the 31 of August have been gathered. Second, the selected models are employed to the series. Then the performance measures are estimated for all models. And then using the models a short-term forecast has been obtained and interpreted. Main findings of the study are summaries as follows. Von Bertalanffy model has outperformed to the other models considered in this study. However, exponential model has predicted the total deaths and cases better than the others. Also, exponential model has given the best prediction errors among them regarding to the death and case figures for last months. Only exponential model has produced a short-term forecast of the daily deaths and cases, nearest to the actual figures. Observed data have tended to increase since the last days of August. This could mean that the COVİD-19 threat has reached to a critical stage to crack down on prevention of pandemics spread. Or it could sign the beginning of a second wave of epidemics.

Remaining parts of this paper have been organized as follows. Growth models used in this study are summarized in Section 2. Implementation and interpretation on the covid-19 figures of Turkey are explained in Section 3. Conclusions are discussed in Section 4.

2. METHODOLOGY

There are several mathematical models to analyze and forecast the pandemic figures in the literature. These models can be categorized in time series models, the compartments-based models and the growth models. I have used five growth models for the COVID-19 figures of Turkey in this study because of their simplicity.

2.1 Logistic Model

Verhulst introduced logistic growth equation which is an extension to the exponential model. The Verhulst logistic equation is also referred to in the literature as the Verhulst-Pearl. Verhulst considered that a stable population would have a saturation level characteristic, and forms a numerical upper bound on the growth size for the model [23].

Logistic growth model has a reduced rate of growth as the population approaches the upper bound, called the carrying capacity. For constants $\alpha_0, \alpha_1 \text{ and } \alpha_2$ the logistic growth of a population over time t is represented by the model (1). Where, α_0 is the carrying capacity.

$$g(t) = \alpha_0/(1 - \alpha_1 \exp(-\alpha_2 t)) \quad (1)$$

Jia et al. [14], Chen et al. [18], Roosa et al. [19] and Wu et al. [20] used logistic growth models for Short-term Forecasts of the COVID-19 epidemic.

2.2 Von Bertalanffy's Model

Von Bertalanffy [24] introduced his growth curve model to model fish weight growth. This model is used to model mean length depending on age in animals. Basic form of his growth model is in (2). Where, $\alpha_0, \alpha_1 \text{ and } \alpha_2$ are the constants of the model.

$$g(t) = \alpha_0(1 - \exp(-\alpha_1 t))^{\alpha_2} \quad (2)$$

Where $\alpha_2 = 1$ the model is becoming an exponential model with two parameters as in (3).

$$g(t) = \alpha_0(1 - \exp(-\alpha_1 t)) \quad (3)$$

Jia et al. [14] used Von Bertalanffy's growth models to cumulative cases and cumulative deaths of COVID-19.

2.3 Gaussian Model

The third model I have used is Gaussian model with two parameters which is the special form of the model in (3) as in (4). Where, $\alpha_0 \text{ and } \alpha_1$ are the constants of the model and α_0 represents the capacity.

$$g(t) = \alpha_0(1 - \exp(-(t/\alpha_1)^2)) \quad (4)$$

2.4 Richards Model

Richards growth model is a variant of logistic growth model and fits to S-shaped growth curves [25]. Richards model has been applied in various contexts, which includes ecological settings and also studying the infectious disease transmission. Wu et al. [20] applied Richards model to the cumulative number of cases of the COVID-19 epidemics. The model used in this study is given in (5). Where, α_0, α_1 and α_2 are the constants of the model.

$$g(t) = \alpha_0/(1 + \alpha_1 \exp(-\alpha_1\alpha_2 t))^{1/\alpha_1} \qquad (5)$$

2.5 Estimation of the Parameters

Parameter estimation is an important step of model development. Maximum likelihood estimation and least square estimation (LSE) are the most common approaches. In this study, LSE approach is used to estimation of the parameters as in (6). Where, $\hat{\boldsymbol{\alpha}}$ is the estimation of the $\boldsymbol{\alpha}$, $y(t)$ is the observed value at time t, and $\hat{g}(t)$ is the prediction at time t by the considered model $g(t)$.

$$\hat{\boldsymbol{\alpha}} = \arg\min \sum \left(y(t) - \hat{g}(t)\right)^2 \qquad (6)$$

2.6 Validation of the Models

Model validation is another important step to develop a model for outbreaks, pandemic and other growth of biological population before using for forecasting. This can be done estimating some performance metrics. Root Mean Square Error (RMSE), R^2 and AIC are used to compare model performance in this study, and they are given in (7), (8) and (9) respectively.

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y(t) - \hat{g}(t))^2} \qquad (7)$$

$$R^2 = 1 - \sum_{i=1}^{n}(y(t) - \hat{g}(t))^2 / \sum_{i=1}^{n}(y(t) - \bar{y})^2 \qquad (8)$$

$$\mathrm{AIC} = n\ln\left(\sum_{i=1}^{n}(y(t) - \hat{g}(t))^2\right) + 2k \qquad (9)$$

Where k is the number of parameters, n is the number of observations, and $\bar{y}$ is the average value of the observations.

3. RESULTS AND DISCUSSION

3.1 Data

Daily figures of COVID-19 for Turkey have been announced by Minister of Health [7]. The data which is used in this study are the collection them. Fig. 1 shows daily deaths of COVID-19 for Turkey between March 11 and August 26. Fig. 2 shows cumulative death figures. The maximum daily dead figure was on 19 of April. After the 1st of June the daily dead figure has been realized around 20s. At the end of august it is tending to increase. The cumulative death figures have reached to 6370 as of 31 of August.

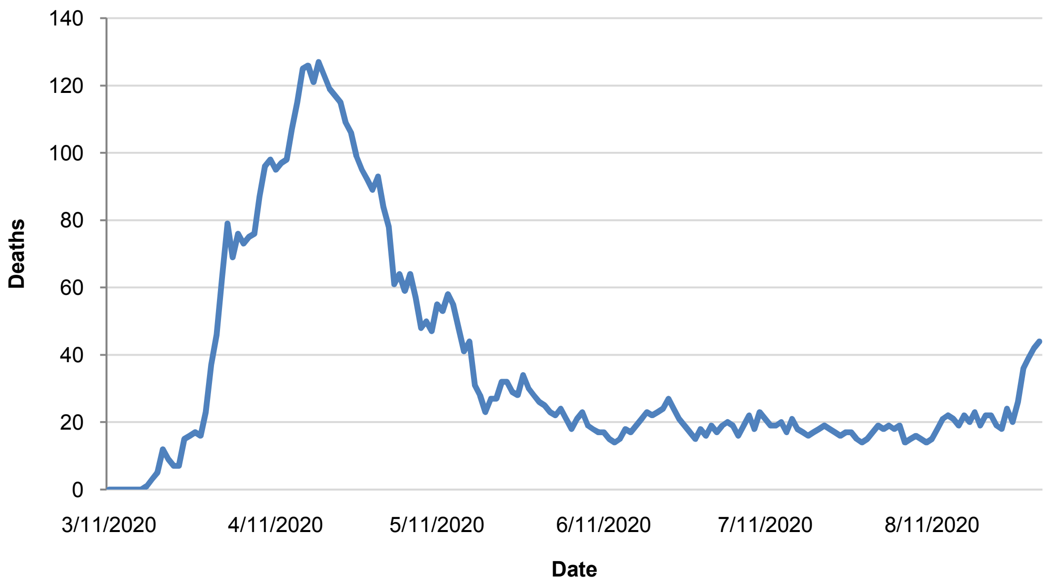

Fig. 1. Daily death figure from COVID-19 for Turkey

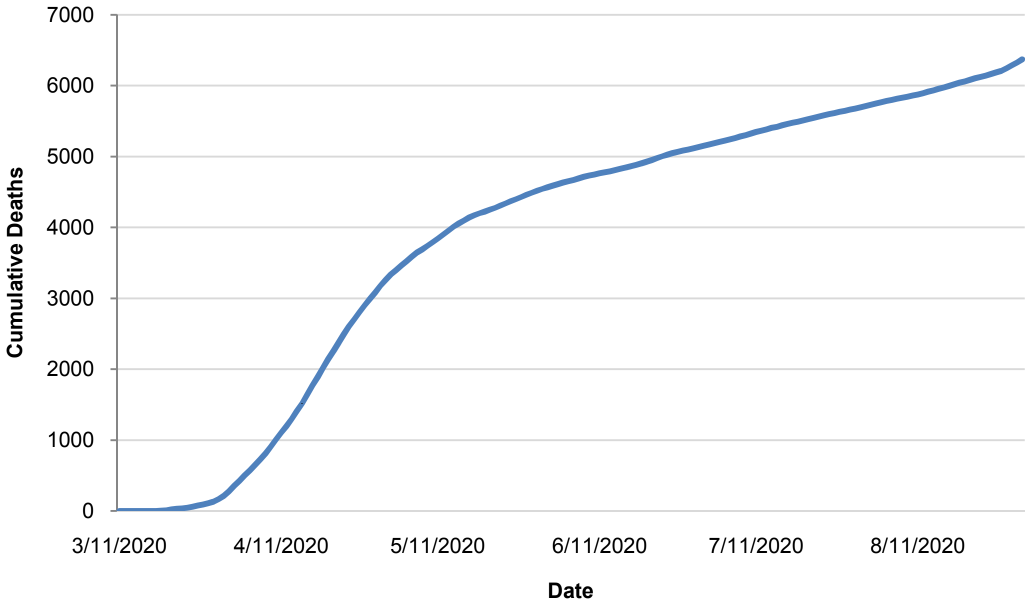

Fig. 2. Cumulative death figure from COVID-19 for Turkey

Fig. 3 shows daily positive cases for Turkey between March 11 and August 31. Fig. 4 shows cumulative case figures. The maximum daily affected cases as 5138 were occurred on 11 of April. After the 1st of June the daily dead figure has been realized around 1000s.

3.2 Results and Discussion

I have estimated the parameters given in Table 1 for all models except Richards model. As mentioned before all growth model considered in this study have a carrying capacity which is α_0 in Table 1. This means that the model can only take value as much as this carrying capacity.

It has been shown that Von Bertanffy, Gaussian, logistic, and exponential growth models fit the cumulative measurement series according to all performance measures in Table 2, respectively. Von Bertanffy growth model forecasted the maximum deaths as 5949 and as of August 31 the predicted deaths as 5806. However, this is approximately % 8,85 less than the actual value, 6370. This shows that the model more fitted to the series at the early stage of the COVID-19. Similar interpretation can be said for Gaussian and logistic growth models. Their predictions are approximately % 12,21 and % 11,95 less than the actual value, respectively. Exponential model has the lowest performance measure among the others. But it forecasted the maximum deaths as 7292, and predicted deaths as 6258 as of 31 August. Its prediction is approximately % 1,76 less than the actual value. As seen at Fig. 5 exponential growth model curve more likely fits to the last data of the series than others. The residuals of the considered growth models are given in Fig. 6. Only residuals of exponential growth model have been so closer to zero than the others regarding since mid-June.

According to growth model structure the pandemic had to be increased rapidly till the reach a peak point, then followed slow increment, and finally remained constant without increasing. The number of daily deaths announced has been fluctuating around the 20's on a daily basis since June 1, and an increasing trend is observed as of the end of August.

A short-term forecasting value of the models are given in Table 3. Since the first of September daily deaths have been announced around 50 and increased day by day. However, exponential growth model forecasted as 11, 10 and 9. An average of 22 deaths with 7.81 standard deviations per day occurred in August. Daily deaths announced for 15 days of September have a mean of 55 with 5.9 standard deviations.

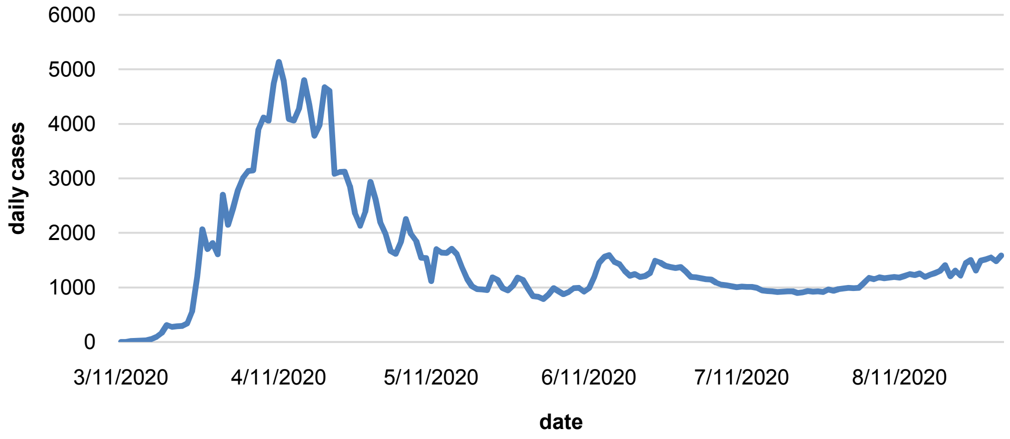

Fig. 3. Daily case figure from COVID-19 for Turkey

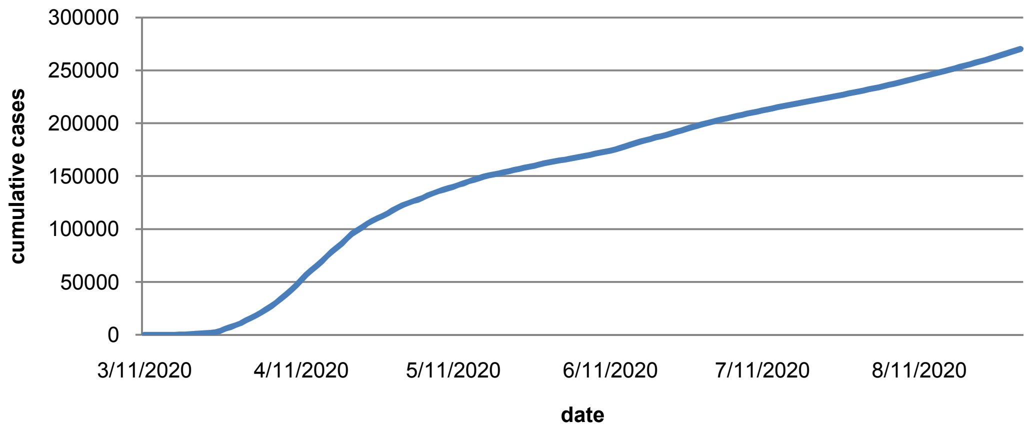

Fig. 4. Cumulative cases of COVID-19 for Turkey

Table 1. Estimated model parameters

Model name	Parameters		
	α_0	α_1	α_2
Logistic	5612.969	-14.693	0.06
Von Bertalanffy	5949.115	0.03	2.333
Exponential	7291.54	0.012	na
Gaussian	5592.664	53.262	na

Table 2. Model performance

Model name	Performance metrics		
	RMSE	R^2	AIC
Logistic	344.87	0.969	2809.3
Von Bertalanffy	203.08	0.989	2632.4
Exponential	352.65	0.967	2815.8
Gaussian	303.1	0.976	2765.2

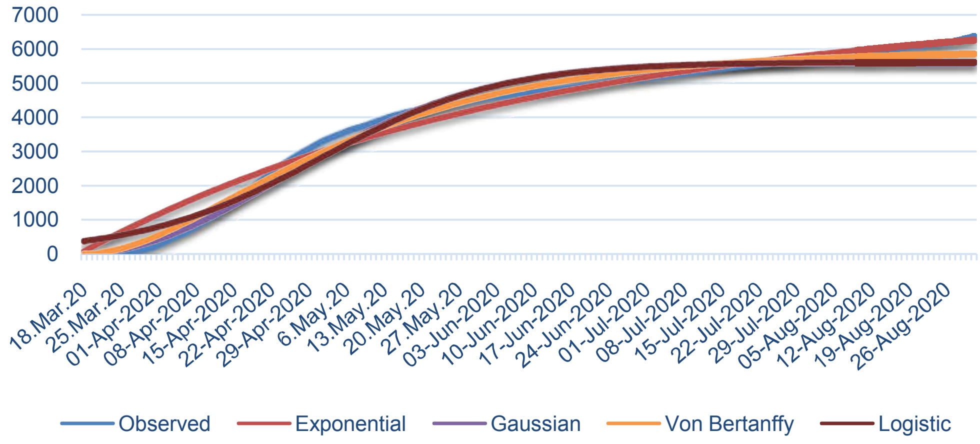

Fig. 5. Fitted curves of the growth models to the cumulative deaths

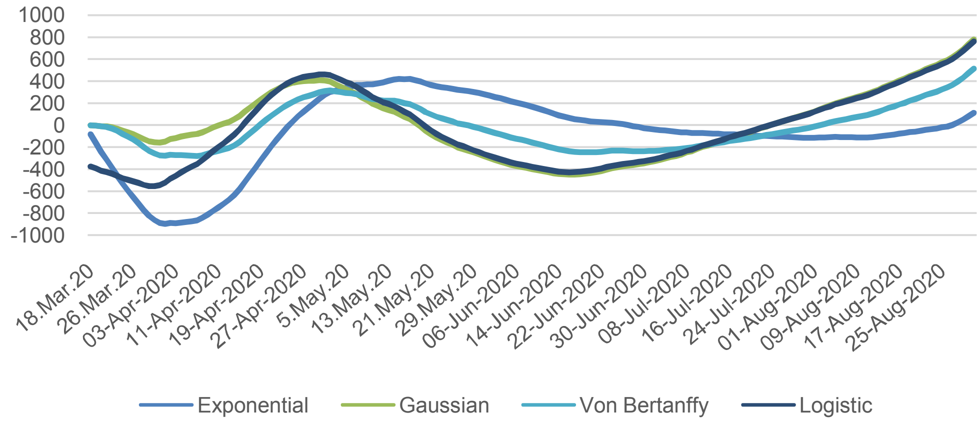

Fig. 6. Residuals of the growth models for the cumulative deaths

Table 3. Short-term forecasting of daily deaths

Day	Logistic	Von Bertalanffy Growth	Exponential Growth	Gaussian	Actual Deaths
1.09.2020	0	2	11	0	47
2.09.2020	0	2	11	0	45
3.09.2020	0	2	11	0	49
4.09.2020	0	2	11	0	53
5.09.2020	0	2	11	0	56
6.09.2020	0	2	10	0	53
7.09.2020	0	2	10	0	52
8.09.2020	0	2	10	0	55
9.09.2020	0	2	10	0	58
10.09.2020	0	2	10	0	56
11.09.2020	0	2	10	0	48
12.09.2020	0	2	10	0	57
13.09.2020	0	2	10	0	63
14.09.2020	0	2	9	0	67
15.09.2020	0	2	9	0	52

Table 4. Estimated model parameters for the cumulative cases

Model name	Parameters		
	α_0	α_1	α_2
Logistic	230750.621	1.948	0.021
Von Bertalanffy	280806.392	0,016	1.573
Exponential	379662.426	0.007	na
Gaussian	232970.237	66.389	na

Table 5. Model performance for the cumulative cases

Model name	Performance metrics		
	RMSE	R^2	AIC
Logistic	40168.8	0.757	4535.034
Von Bertalanffy	9488.9	0.986	4038.654
Exponential	11687.6	0.979	4108.343
Gaussian	16343.3	0.960	4224.699

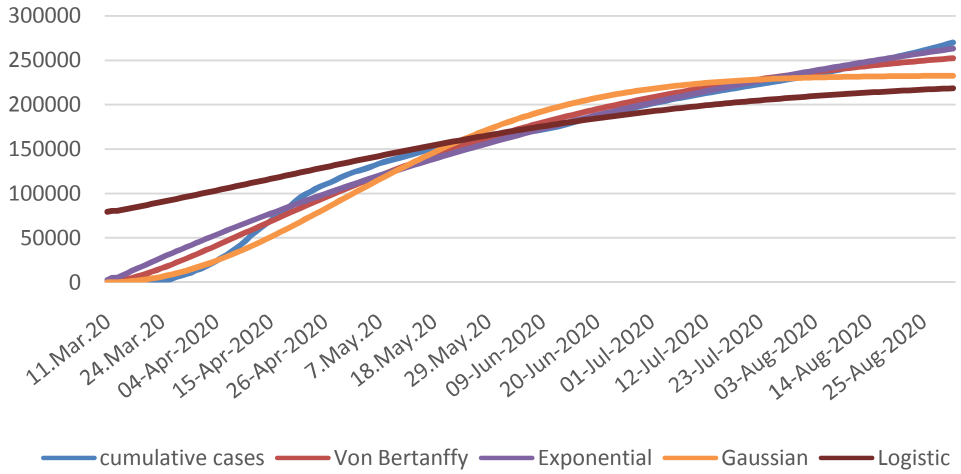

Fig. 7. Fitted curves of the growth models to the cumulative cases

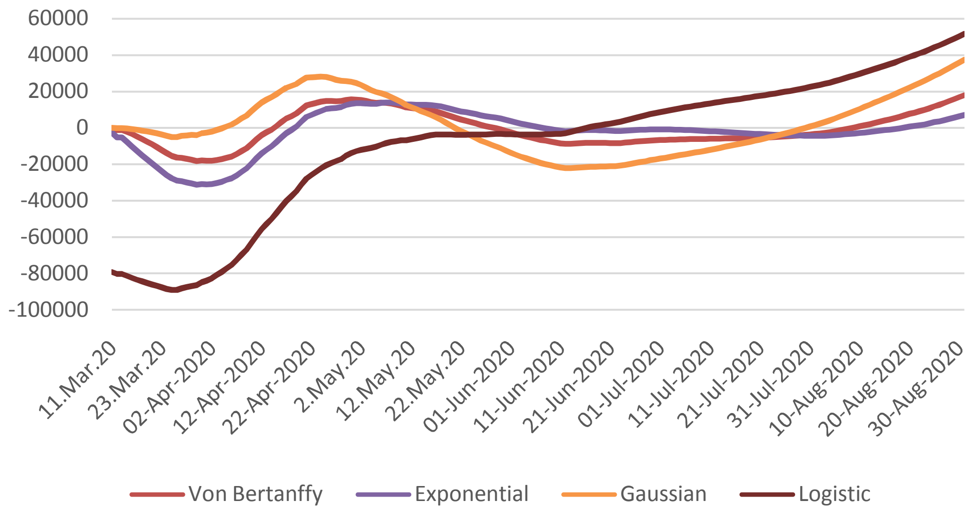

Fig. 8. Residuals of the growth models for the cumulative cases

Table 6. Short-term forecasting of daily cases

Day	Logistic	Von Bertalanffy Growth	Exponential Growth	Gaussian	Actual Deaths
1.09.2020	228	431	794	21	1572
2.09.2020	224	424	789	20	1596
3.09.2020	219	418	783	18	1642
4.09.2020	215	411	778	17	1612
5.09.2020	211	405	773	16	1673
6.09.2020	207	399	767	15	1578
7.09.2020	203	393	762	14	1703
8.09.2020	199	387	756	13	1761
9.09.2020	196	381	751	12	1873
10.09.2020	192	375	746	11	1512
11.09.2020	188	369	741	10	1671
12.09.2020	185	364	736	9	1509
13.09.2020	181	358	730	9	1527
14.09.2020	178	352	725	8	1716
15.09.2020	174	347	720	7	1742

Von Bertanffy, exponential, Gaussian and logistic growth models have fitted the cumulative cases series according to all performance measures in Table 5, respectively. Von Bertanffy growth model forecasted the maximum cumulative cases as 280806 and as of August 31 the predicted deaths as 252182. However, this is approximately % 6.65 less than the actual value, 270633. This shows that the model more fitted to the series at the early stage of the COVID-19. Similar interpretation can be said for Gaussian and logistic growth models. Their predictions are approximately % 13.87 and % 19.11 less than the actual value, respectively. Exponential model is second best model among the others. But it forecasted the maximum deaths as 379663, and predicted deaths as 263045 as of 31 August. Its prediction is approximately % 2.62 less than the actual value. As seen at Fig 7 exponential growth model curve more likely fits to the last data of the series than others. The residuals of the considered growth models are given in Fig. 8. Only residuals of exponential growth model have been so closer to zero than the others regarding since mid-June.

For the daily positive cases, the short-term forecasts value of the models is given in Table 6. Since the first of September daily cases have been announced between 1500 and 1900. However, exponential growth model forecasted nearest value to the actual daily cases. An average of 1267 daily cases with 159.45 standard deviations per day occurred in August. Daily cases announced for 15 days of September have a mean of 1646 with 102.86 standard deviations.

4. CONCLUSION

I have used four growth models to analyze the behavior of pandemic. The death figures from the first of June until at the end of August have fluctuated around 20. An average of 22 deaths with 7.81 standard deviations per day occurred in August. Daily deaths announced for 15 days of September have a mean of 55 with 5.9 standard deviations. However, exponential growth model forecasted as 11, 10 and 9 for 15 days of September. An average of 1267 daily cases with 159.45 standard deviations per day occurred in August. Daily cases announced for 15 days of September have a mean of 1646 with 102.86 standard deviations. Although the Von Bertalanffy model has the best performance measure, it has been observed that the exponential model forecasts the total deaths and total cases better than the others. In addition, exponential growth model has given lower residuals than the other models since mid-June.

The COVID-19 pandemic figures have been increasing tendency since the first of September. This may mean even the first wave has been still continuing by increasing affect or it could sign the beginning of a second wave of epidemics. Several studies must be realized and repeated regularly to learn more about the pandemic when the enough data are available. The data quality is very important for modelling of any pandemics and outbreaks. The early stage of the pandemics modelling studies is strongly biased to the assumptions. So, I have preferred to prepare this study at this time. Growth models are not among the classical time series and forecast methods. These models can be used to investigate the behavior of the epidemic over time, such as when it will reach its peak and when it will end. In addition, it is possible to obtain daily estimates from the model since they are a function of time.

Future studies will be focused on expanding of the study using additional methods on new data.

CONSENT

It is not applicable.

ETHICAL APPROVAL

It is not applicable.

COMPETING INTERESTS

Author has declared that no competing interests exist.

AUTHORS' CONTRIBUTIONS

This work was carried out by Dr. Muzaffer Balaban. Dr. Muzaffer Balaban designed the study, performed the statistical analysis, managed the literature searches, wrote the first draft of the manuscript, read and approved the final manuscript.

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