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A New Mixed Strategy to Cover Continuous Region

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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Abstract

Wireless sensor networks (WSNs) is now a developing area. The coverage problem is an important part of that. WSNs contain sensors which cover a three or two-dimensional bounded convex set. In general, the sensors are deployed randomly from the air. Sometimes sensors are placed by robot(s) or sensor can move to placed themselves on some points of a pre-assigned region, known as Region of Interest (ROI). Since there is a randomness of placing the sensors, they may not be placed on the pre-assigned points. Hence, ROI may not completely cover by the sensors. So, the problem is, how one decrease the uncovered area? Most of the situations, extra sensors are deployed on some randomly chosen points to cover the ROI. In one of our previous work, we reduced the distance between two adjacent vertices and drop exactly one sensor on each vertex. We compare the uncovered area using two strategies (the general one and our previous one), for normal and uniform distributions, and for a different percentages of extra sensors. We observed that our strategy is better for low variance-distribution, but not for high variance-distribution. So, we combined the two strategies in this paper.We partitioned the ROI in regular hexagons and develop a new mixed strategy,

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for dropping the sensors. The extra sensors are divides in two groups. One group is used for reducing the distance between the two adjacent vertices and other parts is used for deploying an extra sensor on randomly selected points. We simulate with a non parametric distribution also along with above two. Simulation shows the optimal trade-off between the percentage of these two groups, which change with the variance of randomness.

Keywords: Coverage problem; random deployment; wireless sensor networks.

1 Introduction

WSNs is a system of a set of sensors. The sensors are small and have a wireless receiver, a processing circuit. Battery capacity of a sensors are low, they have low processing power, they also have a radio. Sensor can measures humidity, direction, distance, temperature, speed, etc. The unique feature of WSNs is that, the sensor can dropped or placed deterministically or randomly in any domain [1]. They provide scope for civilians and military activities. For example, military surveillance, emergency health care, industrial applications, etc.[2]. Abdallah et.al. [3] present placement of WSN for electronics objects in house. Wajgi and Tembhurne [4] present clustering-based localization strategies for WSN in detail. In a wireless multimedia sensor network, clustering is useful to offer distributed computing and parallel processing by grouping the sensor into small subgroups which operate independently.

The main objects of a WSN is observing an event in a region. Hence, covering of the region is necessary for a WSN. To do the job, a WSNs must cover the full region, without any gap. This type of gap is called sensing hole [5]. A sensor can find an object or can detect an event in a disc (called, sensing disc) of a radius (called sensing radius). A point of ROI is not be covered by a set of sensors when the point is not in the sensing radius of any sensor of the WSN. We have to put the sensors in a way, that there should not be any sensing hole. However, one cannot expect that, a sensor will be put at the target point, because sensors are randomly dropped from the helicopter or by a robot [6]. Cause of wrong placement may happen for other factors also. Sensors are deployed in a convex, bounded and closed set of $\mathbb{R}^2$. We must find a strategy for deployment of sensors, so that, the sensing holes will be decreased, if possible, minimized.

We have two distinct methods for sensor placement: (i) placement of sensors in deterministic way and (ii) deploying sensors from the air onto target points [7]. In method (i), it is possible to cover ROI completely by a sufficient number sensors. In method (ii), it is not possible to cover ROI completely even if a sufficient number of sensor are deployed.

When we drop sensors from the air, we may use robots to cover or to decrease the uncovered area of the ROI. This network is known as wireless sensor and actuator network (WSAN). In this network, sensors are randomly or deterministically placed, and after that, the sensors are relocated by robot [8]. In some networks, there is a type of sensor which can move, and can put itself at a neighbouring point without any robot. Since a movable sensor needs large amount of battery capacity or others type of energy, robot assisted sensor placement is more feasible [9, 10].

There are many methods to put sensors efficiently to cover a bounded set of $\mathbb{R}^2$. This problem of covering of a region is usually known as the *coverage problem*. Several variant of coverage problem can be found in [11]. A survey on the strategies of sensor deployment is in [12]. Complete analysis of the expected and the maximum distance covered by a robot, to achieve the full coverage, is in [13, 8]. In previous works, the uncovered area is covered, either by dropping extra sensors or using robot(s), or by activating a group of passive sensors. Nandi et al. describe a new algorithm for the robot to decrease the uncovered region [8].

2 Motivations

Now the important question is 'Is ROI completely covered or not?' and, if the ROI is not completely covered; then how we cover the region using extra sensors. Many works have been done on this, but only few works have been done on the problems: (i) 'How the proportion of the uncovered area is changed w.r.t. the number of sensors?' and (ii) 'How the proportion of the uncovered area depends on the strategies of random placements of the sensors?'.

Note that it is not guaranteed that the region is completely covered even if one use more extra sensors, unless one relocates of those sensors, either using movable sensors or with the help of robot(s). We consider there is no movable sensor and no actuator. Our prime goal is to develop a mixed strategy for deployment of sensors for decrease the uncovered proportion.

Clearly, each point of ROI is sufficient to cover by exactly one sensor. So, if any point of ROI is covered by greater than one sensor; then it is a wastage of sensors. Since the covered area by a sensor is a circular disc, we cannot avoid this wastage. Hence, our goal is to decrease the wastage part of the ROI. One idea is, place the sensors in a deterministic way on the pre-fixed points of ROI, so that, if they are actually put on those points, in that situation, there is minimum wastage. But after the deployment, there must be some uncovered portion for random deployment of the sensors. That is why we need extra sensors.

So, our problem can be restated as follows, how do we placed the sensors so the wastage is decrease or may be minimum, i.e., in which method, we use the extra sensors to decrease the uncovered points or area. Nandi et.al. develop a strategy to solve the above problem in $\mathbb{R}^2$ [14] and Nandi develop a strategy to solve the above problem in $\mathbb{R}^3$ [15]. In above two papers, we assumed that a sensor can be placed at any arbitrary point of ROI. We also assumed that, the distance between two points; where the sensor actually placed; and the target point where we want to place; is random in nature. Now when we placed an extra sensor at few randomly chosen points of ROI, then uncovered area will be reduced. On the other hand, if we placed exactly 1 sensor at the target point, but decrease the distance between two neighbouring target points, then also the uncovered points or area will be decrease. In either situation we keep the sensing radius unchanged. That is, we use some extra sensors in 2 different ways in different two strategies. The differences between the above two are as follows:

- In the first strategy (call it S), we target to deploy 2 sensors on few randomly chosen vertices and 1 sensor onto other vertices. In second strategy (call it S'), we drop exactly 1 sensor onto each vertex.
- Let in S, there are m regular hexagons and l extra sensors, i.e., total $m + l$ sensors is placed in S. If the sides is b in S, then in S' the side will be c where, $(m + l)c^2 = mb^2$. Hence the total covered area is equal in either case. The distance between two target vertices is less for S' than that of S.

In the above 2 papers, we consider the ROI is a convex bounded subsets of $\mathbb{R}^2$ or $\mathbb{R}^3$. The distance (D_i) between two aforesaid points is a random variable. We assumed probability distribution of (D_i) is uniform and may be normal. We simulated the uncovered area for above 2 distributions and for above 2 strategies.

To solve the coverage problem in 2-dimensions, we divide the ROI into required number of regular hexagon. It is well-known that, in the case of deterministic placement, coverage is best when we divide ROI into regular hexagons. But in case of random deployment, the hexagonal division may not be best among all types of partition. In the above two papers, we consider the hexagonal and cube centered partition in case of 2 and 3 dimensions respectively. We have shown that the S is better for distribution which have higher standard deviation and the S' is better for distribution which has smaller standard deviation.

Since the lower and higher standard deviation (s.d.) distribution behaves oppositely for 2 strategies, we hope that in case of moderate (i.e., between higher and lower) s.d. mixing of the above two strategies may be better. In this new mixed strategy, the idea is; divide all extra sensors in two parts; sensors of the first part is used for reducing the distance between the two adjacent vertices (i.e., using our previous method) and then remain sensors is used for dropping two sensors on randomly selected vertices (i.e., using usual method). The percentage of division into two parts depends on the s.d., and hope that there is a optimal division of two parts depending on the corresponding s.d. Here we divide the ROI into regular hexagon. Simulation results show our hypotheses is correct. Results also suggest the optimal division of two parts. We consider uniform, normal distributions and a nonparametric distribution.

3 Assumptions and Definitions

Formally we can state the problem as follows: for an indexed set T, let the set of unit discs $\{C_T \subseteq \mathbb{R}^2 : t \in T\}$, which can cover a set in $\mathbb{R}^2$. This set to be considered as ROI. Let collection of two-dimensional random vectors be $\{Y_t : t \in T\}$. Let D_t be the distance between Y_t and the center of C_t, for $t \in T$. Assume that D_t's are i.i.d. We consider three probability distributions for D_t here, one is nonparametric and other two are uniform and normal.

Assume the ROI is partitioned into similar regular hexagons of length b. To cover each regular hexagon by one sensor, we have to choose $b \leq r$, where r is the sensing radius. If $r = b$ each hexagon will be completely covered by a sensor if the sensor is put on the center of the corresponding hexagon. Since a sensor is too small, so one can assume sensor as a point.

Let there be n regular hexagons and l extra sensors, that is, overall $n + l$ sensors is used. Let $k(< l)$ sensors are used for reducing distance between 2 adjacent vertices and $l - k$ sensors are used for deploying 2 sensors on $l - k$ randomly chosen vertices. If the initial length of the sides is a, then the length of the side will be changed to b where $(n + k)b^2 = na^2$. Hence overall covered area will be same for both. Now we define important concepts and terms. Some terms are defined in [16, 17].

- *Sensing Disc* T_M of a sensor M is a circular disc of radius r with the center at M, which is covered by the sensor placed at the corresponding node. The radius of the circular disc, r is known as *sensing radius*. Sensing radius is assumed to be same for all discs.
- $N(V)$ is the node which corresponds to a vertex V, that is, a sensor is placed on $N(V)$ but the target was to place at V. $V(N)$ is the respective vertex of a node N.
- $\mathcal{V}$ is set of all vertices and Adj_V is set of all the adjacent vertices of vertex V. Similar definitions and notations can be formulated for a sensor also. The respective notations are $\mathcal{M}$ and Adj_M for $M \in \mathcal{M}$.
- $d(R, S)$ be the distance between two points R and T.
- A point $P \in \mathbb{R}^n$ is defined to be covered by sensor M if the distance between P and M, $d(P, M) \leq r$ and P is said to be covered by a set of sensors $\mathcal{M}$ if P is covered by at least one sensor in $\mathcal{M}$. A point $P \in \mathbb{R}^n$ is said to be uncovered by a node M if it not covered by M and the point P is said to be not covered or uncovered by $\mathcal{M}$ if P is not covered by sensors in $\mathcal{M}$.
- Area of a set T will be denoted by $\mathrm{A}(T)$.

Observe that if a sensor is placed on the target, then the vertex and its corresponding node is equal, i.e., $V(N) = N$ and $N(V) = V$.

Now we define the concept 'wastage'. Let T be any convex set in $\mathbb{R}^2$, which is covered by a finite (or infinite) set of sensors $\mathcal{M}$. The wastage in T for $\mathcal{M}$ is define as follows:

$$W_{\mathcal{M}}(T) = \frac{\sum\limits_{M \in \mathcal{M}} \mathrm{A}(T \cap T_M) - \mathrm{A}(T)}{\sum\limits_{M \in \mathcal{M}} \mathrm{A}(T \cap T_M)} \cdots (1).$$

If $\mathcal{M}$ is a set so that $|T_{M_1} \cap T_{M_2} \cap S_{M_3}| \leq 1$ for distinct $M_1, M_2, M_3 \in \mathcal{M}$, then

$$W_{\mathcal{M}}(T) = \frac{\sum\limits_{M_1 \neq M_2 \in \mathcal{M}} \mathrm{A}(T \cap T_{M_1} \cap T_{M_2})}{\sum\limits_{M \in \mathcal{M}} \mathrm{A}(T \cap T_M)} \cdots (2).$$

Intuitively, the denominator of is the overall area, which intersect with T, and the numerator is the wastage area in common sense. Here wastage is actually the proportion of wastage area with respect to the overall area.

Let $\mathcal{M}$ be a set of sensors which cover $\mathbb{R}^2$. Let $\mathcal{M} \cap T$ is a finite set for all bounded set T. Definition of wastage in $\mathbb{R}^2$ for $\mathcal{M}$ is

$$W_{\mathcal{M}}(\mathbb{R}^2) = \lim_{y \to \infty} W_{\mathcal{M} \cap B_y}(B_y), \cdots (3)$$

where B_y is the disc in $\mathbb{R}^2$ with radius y and center at origin.

Intuitively, wastage in $\mathbb{R}^2$ is the ratio of the wastage area in $\mathbb{R}^2$. Note that we can take any non-decreasing sequence of sets whose limit set (i.e., the union of all sets) is $\mathbb{R}^2$ and we can define wastage in a similar manner. Prove of the result that, two definitions are equivalent, is given in any standard book in measure theory.

4 Simulation Result

Now we describe the simulation technique in details and provide the data, what we get from the simulations. We used C-code for simulation.

Table 1. Simulation result of proportion of the coverage area for $U(0.5)$

$p_1 \rightarrow$	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.00	0.97090	0.97103	0.97371	0.97811	0.98251	0.98502	0.98527	0.98642	0.98831	0.98890	0.99150
0.05	0.96901	0.97392	0.97734	0.98012	0.98521	0.98585	0.98641	0.98841	0.98907	0.99155	0.99159
0.10	0.97590	0.97712	0.97998	0.98531	0.98618	0.98651	0.98854	0.98920	0.99156	0.99161	0.99173
0.15	0.97752	0.97976	0.98582	0.98681	0.98690	0.98860	0.98952	0.99159	0.99165	0.99182	0.99202
0.20	0.98112	0.98601	0.98729	0.98695	0.98862	0.98982	0.99161	0.99168	0.99190	0.99211	0.99359
0.25	0.98527	0.98829	0.98892	0.98893	0.98990	0.99160	0.99169	0.99194	0.99243	0.99361	0.99447
0.30	0.98789	0.98875	0.98897	0.98991	0.99161	0.99172	0.99197	0.99256	0.99364	0.99453	0.99529
0.35	0.98865	0.98998	0.99021	0.99168	0.99178	0.99202	0.99298	0.99370	0.99458	0.99531	0.99572
0.40	0.98992	0.99096	0.99171	0.99180	0.99200	0.99301	0.99378	0.99462	0.99538	0.99587	0.99651
0.45	0.99098	0.99180	0.99185	0.99200	0.99300	0.99389	0.99461	0.99540	0.99597	0.99686	0.99701
0.50	0.99179	0.99189	0.99201	0.99312	0.99397	0.99477	0.99541	0.99601	0.99699	0.99741	0.99800

Table 2. Simulation result of proportion of the coverage area for $U(1.0)$

$p_1 \rightarrow$	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.00	0.92341	0.92918	0.93350	0.93680	0.94500	0.94876	0.95321	0.95676	0.95753	0.95932	0.96247
0.05	0.92904	0.93412	0.93762	0.94519	0.94901	0.95410	0.95782	0.95799	0.96071	0.96350	0.96765
0.10	0.93380	0.93942	0.94597	0.94998	0.95512	0.95802	0.95843	0.96226	0.96454	0.96854	0.97139
0.15	0.94231	0.94625	0.95016	0.95601	0.95897	0.95957	0.96352	0.96589	0.96932	0.97199	0.97731
0.20	0.94601	0.95321	0.95754	0.95964	0.96187	0.96432	0.96672	0.97031	0.97267	0.97719	0.98001
0.25	0.95676	0.95807	0.96002	0.96352	0.96517	0.96749	0.97110	0.97318	0.97801	0.98062	0.98236
0.30	0.95997	0.96187	0.96529	0.96602	0.96831	0.97192	0.97400	0.97893	0.98135	0.98311	0.98471
0.35	0.96302	0.96754	0.96725	0.96932	0.97275	0.97532	0.97971	0.98211	0.98389	0.98542	0.98666
0.40	0.96896	0.96843	0.97076	0.97327	0.97687	0.98056	0.98301	0.98467	0.98608	0.98732	0.98831
0.45	0.97003	0.97197	0.97401	0.97802	0.98148	0.98399	0.98542	0.98692	0.98809	0.98902	0.99001
0.50	0.97300	0.97521	0.97901	0.98231	0.98489	0.98611	0.98755	0.98897	0.98999	0.99053	0.99102

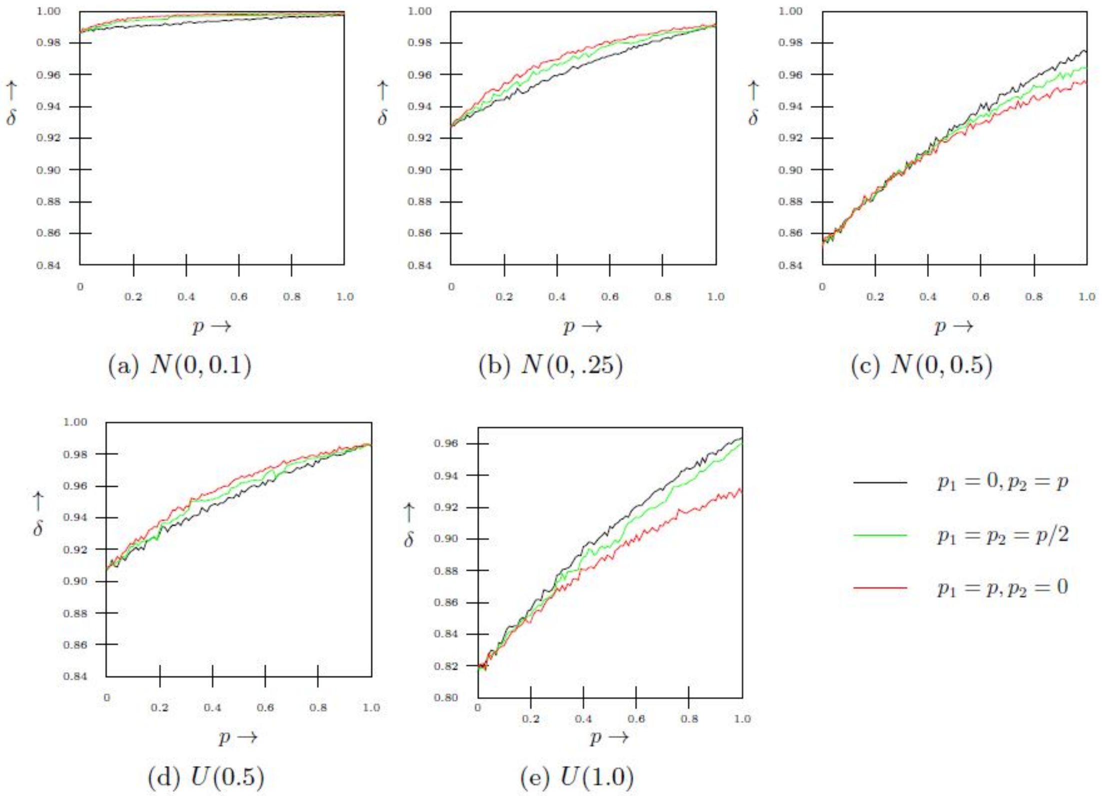

Fig. 1. Graph (from simulation data) of proportion of coverage area in $\mathbb{R}^2$

Clearly radius of sensing disc (i.e., r) has no importance in the simulation. In our simulation, we consider 10000 nodes with $r = 1$. Then we divide the ROI in to congruent regular hexagons and target to drop exactly one sensor onto the center of these regular hexagons. The overall area is $10000 \times \frac{3\sqrt{3}}{2}$ unit, and distance between adjacent vertices is $\sqrt{3}$ unit. We put 100 sensors in a row and there are 100 rows. We simulate the uncovered area considering the distances D_i's between the targeted vertex and the point where it is placed are i.i.d. We simulate with three different distributions uniform, normal and a non-parametric. The p.d.f. of uniform distribution $U(t)$ is $f(x) = \frac{2x}{t^2} I_{(0,t)}$. Normal distribution with mean 0 and s.d. t is denoted by $N(0, t^2)$. The non-parametric distribution is generated randomly by simulation.

Let $(p_1+p_2)\%$ more sensors are used, hence the number of sensors is $10000 \times \left(1 + \frac{p_1+p_2}{100}\right)$ where $(p_1+p_2) \in [0, 100]$. Our new mixed strategy is as below:

Partition ROI into $10000\left(1 + \frac{p_1}{100}\right)$ regular hexagon of length $\sqrt{\frac{100}{100+p_1}}$. Consider $10000\left(1 + \frac{p_1}{100}\right)$ hexagons and their centers as vertices and deploy one sensor exactly for every vertices. Observe that the area of the whole region is $10000\left(1 + \frac{p_1}{100}\right) \times \frac{3\sqrt{3}}{2}\left(\sqrt{\frac{100}{100+p_1}}\right)^2 = 10000 \times \frac{3\sqrt{3}}{2} \cdots (4)$.

Next pick $100p_2$ vertices uniformly and randomly from the $10000\left(1 + \frac{p_1}{100}\right)$ and deploy two sensors on each of those vertices and drop exactly one sensor on other $(10000 + 100p_1 - 100p_2)$ vertices. Then we compute the ratio of uncovered area of ROI with different values of p_1, p_2 and t for uniform and normal distribution using

Table 3. Simulation result of proportion of the coverage area for $N(0, 0.1)$

$p_1 \rightarrow$	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.00	0.96569	0.96881	0.97079	0.97591	0.97649	0.97870	0.97976	0.98231	0.98421	0.98520	0.98694
0.05	0.96849	0.97021	0.97542	0.97772	0.97983	0.98001	0.98341	0.98534	0.98597	0.98705	0.98721
0.10	0.96951	0.97387	0.97875	0.98099	0.98081	0.98403	0.98607	0.98653	0.98749	0.98897	0.98911
0.15	0.97230	0.97997	0.98189	0.98205	0.98571	0.98701	0.98712	0.98764	0.98812	0.99001	0.99119
0.20	0.97968	0.98264	0.98310	0.98610	0.98826	0.99839	0.99841	0.98901	0.99100	0.99197	0.99208
0.25	0.98230	0.98421	0.98735	0.98921	0.98953	0.98960	0.98998	0.99241	0.99243	0.99241	0.99294
0.30	0.98543	0.98812	0.99033	0.99034	0.99035	0.99058	0.99098	0.99241	0.99278	0.99307	0.99452
0.35	0.98901	0.99021	0.99035	0.99036	0.99081	0.99149	0.99258	0.99301	0.99378	0.99498	0.99537
0.40	0.98997	0.99025	0.99041	0.99173	0.99261	0.99309	0.99372	0.99450	0.99501	0.99568	0.99619
0.45	0.99054	0.99060	0.99281	0.99324	0.99380	0.99421	0.99471	0.99513	0.99581	0.99674	0.99765
0.50	0.99132	0.99219	0.99310	0.99388	0.99412	0.99490	0.99541	0.99601	0.99721	0.99804	0.99900

Table 4. Simulation result of proportion of the coverage area for $N(0, 0.25)$

$p_1 \rightarrow$	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.00	0.93848	0.94550	0.94750	0.95048	0.95502	0.95770	0.96321	0.96432	0.96764	0.97001	0.97052
0.05	0.94401	0.94732	0.95164	0.95709	0.95887	0.96398	0.96444	0.96814	0.97136	0.97156	0.97290
0.10	0.94700	0.95287	0.95851	0.95998	0.96405	0.96453	0.96887	0.97202	0.97245	0.97378	0.97463
0.15	0.95402	0.96021	0.96169	0.96483	0.96488	0.96853	0.97303	0.97343	0.97456	0.97524	0.97595
0.20	0.96184	0.96301	0.96557	0.96500	0.96961	0.97387	0.97456	0.97532	0.97609	0.9673	0.97747
0.25	0.96452	0.96638	0.96574	0.96050	0.97467	0.97572	0.97614	0.97692	0.97737	0.97831	0.97931
0.30	0.96712	0.96621	0.97153	0.97542	0.97654	0.97699	0.97769	0.97829	0.97907	0.97989	0.98042
0.35	0.96678	0.97128	0.97621	0.97731	0.97786	0.97852	0.97901	0.97982	0.98075	0.98134	0.98196
0.40	0.97055	0.97532	0.97802	0.97864	0.97932	0.97980	0.98051	0.98163	0.98222	0.98273	0.98328
0.45	0.97425	0.97898	0.97976	0.98021	0.98074	0.98163	0.98257	0.98301	0.98356	0.98414	0.98532
0.50	0.97873	0.97965	0.98045	0.98134	0.98245	0.98321	0.98389	0.98434	0.98509	0.98600	0.98623

simulation, where p_1, p_2, t are real numbers between 0 and 1. We repeat the whole procedure for a set of three parameters p_1, p_2 and t for 10000 times and take the average of those 10000 ratios. Then we repeat the steps for different sets of parameters. Next, we find the average of ratios for different sets of three parameters. Again, we do the whole simulation for uniform, normal and a non-parametric distribution.

We have the following Tables 1 to 6 showing the proportion of the uncovered area of ROI (written in the body) with different values of p_1, p_2 for $U(0.5), U(1.0), N(0, 0.1)$ and $N(0, 0.25)$ respectively. Here the first row represent the value of p_1 and first column represent the value of p_2.

From the tables we can find the best choice of the p_1 and p_2 for fixed value of p with $p = p_1 + p_2$. As for example, consider the distribution $U(0.5)$ below:

For $p = 0.05$ when $p_1 = 0$ and $p_2 = 0.05$ the average ratio of the covered area is 0.96901, whereas when $p_1 = 0.05$ and $p_2 = 0$ the average ratio of the covered area is 0.97103. So, we can conclude that, the second choice is better

Table 5. Simulation result of proportion of the coverage area for $N(0, 0.5)$

$p_1 \rightarrow$	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.00	0.92843	0.93503	0.93502	0.93948	0.94021	0.94377	0.95132	0.95343	0.95676	0.95900	0.96105
0.05	0.93344	0.93473	0.94051	0.94579	0.95588	0.95639	0.95644	0.95814	0.96071	0.96176	0.96372
0.10	0.93647	0.94152	0.94758	0.94859	0.95364	0.95464	0.95688	0.96072	0.96172	0.96378	0.96574
0.15	0.94454	0.94660	0.94761	0.95264	0.95464	0.95868	0.96473	0.96734	0.96874	0.96875	0.96975
0.20	0.95061	0.95363	0.95655	0.95665	0.96096	0.96473	0.96574	0.96753	0.96876	0.96973	0.97074
0.25	0.95645	0.95866	0.95965	0.96050	0.96774	0.96875	0.96914	0.97069	0.97373	0.97783	0.97951
0.30	0.96071	0.96162	0.96715	0.96875	0.96954	0.97169	0.97376	0.97282	0.97590	0.97698	0.98002
0.35	0.96067	0.96712	0.96762	0.96877	0.96877	0.96978	0.97090	0.97298	0.97580	0.97814	0.98019
0.40	0.96705	0.96753	0.968782	0.96864	0.97093	0.97398	0.97580	0.97816	0.97822	0.97982	0.98032
0.45	0.97025	0.97289	0.97197	0.97802	0.97980	0.98016	0.98025	0.98130	0.98235	0.98341	0.98452
0.50	0.97083	0.97296	0.97384	0.97813	0.97825	0.98032	0.98138	0.98434	0.98309	0.98460	0.98563

Table 6. Simulation result for non-parametric distribution with s.d. 0.5

$p_1 \rightarrow$	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.00	0.92432	0.93032	0.93102	0.93482	0.93640	0.94077	0.94513	0.95043	0.95167	0.95390	0.95610
0.05	0.93044	0.93307	0.93405	0.93998	0.94588	0.95039	0.95432	0.95541	0.95607	0.96076	0.96221
0.10	0.93471	0.93941	0.94275	0.94591	0.94642	0.95046	0.95368	0.95607	0.95861	0.96037	0.96257
0.15	0.94254	0.94366	0.94612	0.95064	0.95146	0.95681	0.96147	0.96347	0.96428	0.96532	0.96750
0.20	0.94615	0.948536	0.95165	0.95266	0.95609	0.95739	0.96157	0.96331	0.96587	0.96737	0.96977
0.25	0.95452	0.95661	0.95796	0.95960	0.96277	0.96487	0.96791	0.96870	0.97137	0.97478	0.97695
0.30	0.95607	0.95861	0.96071	0.96287	0.96495	0.96716	0.97037	0.97182	0.97359	0.97569	0.97802
0.35	0.95706	0.95967	0.96176	0.96387	0.96587	0.96830	0.97000	0.97208	0.97458	0.97681	0.97893
0.40	0.96052	0.96343	0.96587	0.96648	0.96709	0.96818	0.97023	0.97163	0.97382	0.97598	0.97803
0.45	0.96702	0.96899	0.97097	0.97380	0.97598	0.97801	0.97980	0.98092	0.98123	0.98234	0.98401
0.50	0.96783	0.96961	0.97184	0.97339	0.97521	0.97803	0.98038	0.98240	0.98300	0.98346	0.98531

than the first. For $p = 0.25$ when $p_1 = 0.05$ and $p_2 = 0.20$ the average ratio of the covered area is 0.98601, which is the best choice of p_1 and p_2. If we take more values of p_1 and p_2 in the interval, we will get the more precise optimal choice of p_1 and p_2.

Finally, we draw the 'average ratio of covered area (δ)' vs. number of sensors ('p') graphs for 5 distributions and three different values of p_1 and p_2 (see Fig. 1a to 1e). The red, blue and black curves show the values of proportion of covered area for different values of p and for $(p_1, p_2) = (p, 0), (p_1, p_2) = (p/2, p/2)$ and $(p_1, p_2) = (0, p)$ respectively. From the curves we can find the best choice of p_1 and p_2. Note that one can draw more curves for different choices of p_1 and p_2 to get a more precise optimal choice of p_1 and p_2.

5 Conclusion

In our paper, we want to give some new idea in the coverage problem in $\mathbb{R}^2$. We consider the sensors may not be perfectly put on the target, whereas it can be placed at a random point in the region. Our assumption is, the distance between the aforesaid 2 points is random and follow i.i.d. We also consider that distribution to be uniform, normal and nonparametric. For these three distributions we have done simulations using C-code. To decrease the wastage i.e., uncovered area, we develop a new mixed strategy of using sensors. We have compared the new strategy with older one. We consider hexagonal partition of ROI. We divide the extra sensors in 2 different parts. First part is used for reducing the length of hexagon and other parts is used to drop 2 sensors on some randomly selected vertices. Simulation results show our hypotheses is correct. Results also suggest the optimal division of two parts. We consider uniform, normal distributions and a nonparametric distribution.

In future we consider the deployment of sensors for 3-dimensional covering and we may divide ROI into square. We can think other parametic distributions like multivariate normal, exponential distribution for deployment. Here we consider a new mixed strategy to deploy sensors, which is better among all existence strategies. One can find other strategies, which may be better, for distributions other than normal and uniform. In future one may use extra sensors as well as robots.

Competing Interests

Author has declared that no competing interests exist.

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