



Natural Convective Heat Transfer in a Laminar Flow over an Immersed Curved Surface

Charles Mwaniki Nyaga^{1*}, Mary Wainaina¹ and Osman Shaibu¹

¹*Department of Mathematics and Actuarial Science, Faculty of Science, Catholic University of Eastern Africa, P.O.Box 62157-00200, Nairobi, Kenya.*

Authors' contributions

This work was carried out in collaboration between all authors. Author CMN designed the study, wrote the protocol, and wrote the first draft of the manuscript. Author MW managed the analyses of the study. Author OS managed the literature searches. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/PSIJ/2018/43567

Editor(s):

(1) Dr. Humaira Yasmin, Assistant Professor, Department of Mathematics, College of Science, Majmaah University, Saudi Arabia.

(2) Dr. Roberto Oscar Aquilano, School of Exact Science, National University of Rosario (UNR), Rosario, Physics Institute (IFIR)(CONICET-UNR), Argentina.

Reviewers:

(1) Prasun Dutta, Indian Institute of Engineering Science & Technology, India.

(2) Subramaniam Jahanadan, Labuan Matriculation College, Malaysia.

(3) Obiekea Kenneth Nnamdi, Ahmadu Bello University, Nigeria.

Complete Peer review History: <http://www.sciencedomain.org/review-history/26156>

Original Research Article

Received 27th June 2018
Accepted 3rd September 2018
Published 8th September 2018

ABSTRACT

Numerical solutions of unsteady laminar free flow of a viscous fluid past an immersed curved surface were presented in this research study. The two-dimensional fluid flow in consideration was unsteady and incompressible. Flows of this nature are commonly encountered in engineering studies such as Aerodynamics and Hydrodynamics. In our study, the continuity, the momentum and thermal energy equations were non-dimensionalized and the solutions of the dimensionless governing equations approximated using finite-difference method. The velocity and temperature fields were studied by varying various parameters in the equations governing the fluid flow. The results obtained were presented graphically for comprehensive and easier interpretation. From the results, it was found out that the dissipation of heat within the boundary layer increases with increase in the length of the curvature i.e. when the length of the curvature was increased, a consequence increase in the amount of heat dissipated within the boundary layer was noted. Also at large Reynolds number, minimal amount of heat dissipated within the boundary layer was recorded. These findings would assist Engineers in making appropriate designs and estimate improvements in equipment that require minimal resistance to the fluid in motion.

*Corresponding author: E-mail: charlesnyaga2@gmail.com;

Keywords: *Incompressible viscous fluid; velocity field; temperature field.*

1. BACKGROUND INFORMATION

Natural Convective heat transfer over an immersed curved surface is receiving research attention due to its wide applications in designing of devices such as flying planes, submarines, pumps, cooling fans among many others [1].

In the study of laminar flows Gupta et al. [2] investigated heat transfer along the surface with a longitudinal curvature and concluded that as the curvature changes from concave to convex, the Nusselt number decreases for Eckert number being small and increases if the Eckert number is increased to unity.

Bradshaw et al. [3] and Gordon et al. [4] extended the study on the use of the algebraic analogy to the curved shear layers and the effects of the curvature on the mixing length if the shear layer thickness exceeds 1/300 of the radius of the curvature. In their study they concluded that large effects occurred in compressible fluid flows.

Mugambi et al. [5] in their investigation on the forces produced by the fluid motion on a submerged finite curved plates established the relationship between geometrical shape of the curvature and the variation of drag force of specific velocities of the viscous fluid.

George et al. [6] and Barenblatt et al. [7] in their study on the convective heat transfer over curved surface established that as fluid flows over an immersed curved surface, some work is done

against viscous effect and energy spent is converted into heat. The vortices formed in the boundary layer due to high velocity gradient is swept outwards from the boundary layer. They established that the rate of heat transfer is considerably high at points close to the convex surface within the boundary layer thickness. This, as a result, leads to a decrease in fluid viscosity.

Gathungu [1] and Fukagata et al. [8] in their study noted that when the Reynolds number is high, the heat dissipation in the boundary layer also goes high. Their study concluded that when the Reynolds number is increased, the consequence is decreased in drag. When the Reynolds number decreased, the effect of drag goes high. At high Reynolds number the lift is increased and vice versa, hence a direct proportionality of the two quantities.

From the above- discussed research investigations and findings, it is clear that limited or little attention has been paid on the extent to which varying the length of the curvature would affect the velocity and temperature profiles along the unsteady laminar fluid flow. This is the motivation of this research work.

2. MATHEMATICAL FORMULATION

In this research work, a two-dimensional laminar unsteady flow of a fluid over an immersed curved surface is studied. Since the body had both convex and concave surfaces there existed two non- zero pressure gradients as shown in the schematic diagram below.

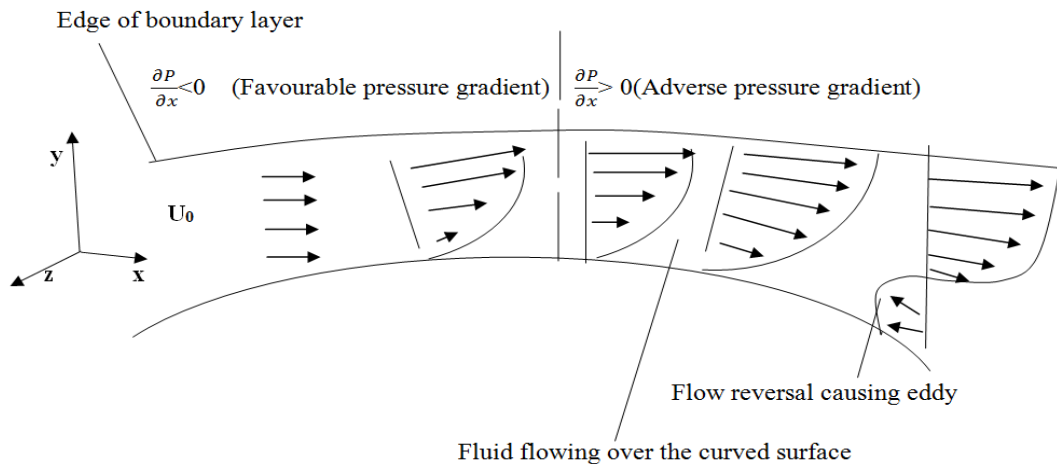


Fig. 1. Schematic model for the flow geometry

2.1 Equation of Continuity

The general continuity equation is given as:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0 \quad (1)$$

For two-dimensional fluid flow with constant density, equation (1.0) reduces to:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1.1)$$

2.2 Momentum Equation

Along the x-axis;

$$\rho \frac{\partial u}{\partial t} + \rho \left[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial x} \left\{ \mu \left[2 \frac{\partial u}{\partial x} - \frac{2}{3} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right] \right\} + \frac{\partial}{\partial y} \left\{ \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right\} + \rho F_x$$

Along the y- axis;

$$\rho \frac{\partial v}{\partial t} + \rho \left[u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = -\frac{\partial P}{\partial y} + \frac{\partial}{\partial y} \left\{ \mu \left[2 \frac{\partial v}{\partial y} - \frac{2}{3} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right] \right\} + \frac{\partial}{\partial x} \left\{ \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right\} + \rho F_y$$

Since $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$, equations the above equations reduces to:

$$\rho \frac{\partial u}{\partial t} + \rho \left[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = -\frac{\partial P}{\partial x} + 2\mu \frac{\partial^2 u}{\partial x^2} + \mu \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y} \right) + \rho F_x \quad (1.2a)$$

and

$$\rho \frac{\partial v}{\partial t} + \rho \left[u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = -\frac{\partial P}{\partial y} + 2\mu \frac{\partial^2 v}{\partial x^2} + \mu \left(\frac{\partial^2 u}{\partial y \partial x} + \frac{\partial^2 v}{\partial y^2} \right) + \rho F_y \quad (1.2b)$$

respectively.

From the boundary layer approximations, equation (1.2a) reduces to:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2} + F_x$$

But $\frac{\mu}{\rho} = \nu$ and thus the above equation further reduces to:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} + F_x \quad (1.3a)$$

Also, equation (1.2b) reduces to:

$$0 = -\frac{1}{\rho} \frac{\partial P}{\partial y} + F_y \quad (1.3b)$$

From Bernoulli's equations, we have

$$P + \frac{1}{2} \rho u^2 = \text{constant} \quad (1.4)$$

The curved surfaces provide both adverse and favourable pressure gradients whose tangential components of the velocity of the outer flow reveals a power law dependence on the stream wise x measured along the curved surface boundary as;

$$\frac{u}{c} = x^m \tag{1.5}$$

Differentiating partially equation (1.4) with respect to x, we obtain

$$\frac{\partial P}{\partial x} + \rho u \frac{\partial u}{\partial x} = 0 \tag{1.6}$$

Which implied that;

$$-\frac{1}{\rho} \frac{\partial P}{\partial x} = u \frac{\partial u}{\partial x} \tag{1.7}$$

But from the power law dependence,

$$u \frac{\partial u}{\partial x} = mc^2 x^{2m-1} \tag{1.8}$$

Hence equation (1.3a) reduces to;

$$\frac{\partial u}{\partial t} = P_t + v \frac{\partial^2 u}{\partial y^2} + F_x \text{ where } P_t = m c^2 x^{2m-1} \tag{1.9}$$

Prandtl proposed to account for curvature effect by multiplying the length of the curvature by factor f given by:

$$f = -\frac{1}{4} \frac{k_r u}{\left(\frac{\partial u}{\partial y}\right)} + 1 \tag{2}$$

He also deduced that the boundary layer equation on the curved surface is written as;

$$\rho k_r u^2 = h_1 \frac{\partial P}{\partial y}, \text{ which is re-written as}$$

$$\frac{1}{\rho} \frac{\partial P}{\partial y} = \frac{k_r u^2}{h_1} \tag{2.1}$$

Where k_r and h_1 are curvature parameters which are defined as

$$Kr = -\frac{1}{c(x)}$$

$$h_1 = 1 + k_r y$$

Where $c(x)$ is the radius of the curvature.

Body forces, F_x and F_y due to the gravitational pull are assumed to be a constant in both cases and thus the assumption:

$$F_x = F_y \tag{2.2}$$

Hence the generalized equation of conservation of momentum for fluid flow over an immersed curved surface is derived as;

$$\frac{\partial u}{\partial t} = P_t + v \frac{\partial^2 u}{\partial y^2} + \frac{k_r u^2}{h_1} \tag{2.3}$$

Since $h_1 = 1 + k_r y$, the term $\frac{k_r u^2}{h_1}$ is written in Taylor series as

$$k_r u^2 (1 + k_r y)^{-1} = k_r u^2 (1 - k_r y + k_r^2 y^2 + \dots)$$

And therefore, equation (2.3) yields

$$\frac{\partial u}{\partial t} = P_t + v \frac{\partial^2 u}{\partial y^2} + k_r u^2 (1 - k_r y + k_r^2 y^2 + \dots) \tag{2.4}$$

The flow is along the x- axis. This implies that $y \cong 0$ and for every small value of k_r , we have $(1 - k_r y + k_r^2 y^2 + \dots) = 1$. Consequently, equation (2.4) reduces to

$$\frac{\partial u}{\partial t} = P_t + v \frac{\partial^2 u}{\partial y^2} + k_r u^2 \tag{2.5}$$

This is our momentum equation in consideration

2.3 The Energy Equation

The general equation is given as

$$\rho C_p \frac{Dh}{Dt} = K \nabla^2 T + \mu \phi, \tag{2.6}$$

Where

$$\phi = 2 \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 \right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 \tag{2.7}$$

By considering unsteady incompressible flow in a control volume, the standard thermal energy equation for the thermal boundary layer is given by

$$\rho v \frac{\partial h}{\partial y} + \rho \frac{\partial h}{\partial t} + \rho u \frac{\partial h}{\partial x} = (\mu \phi + q) + \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \left(u \frac{\partial P}{\partial x} + v \frac{\partial P}{\partial y} \right) \tag{2.8}$$

where h was the enthalpy and q was the rate of heat dissipation.

Now the enthalpy h is given by:

$$h = E + P \left(\frac{1}{\rho}\right) \tag{2.9}$$

then, the first order derivative of enthalpy becomes

$$dh = dE + \left(\frac{1}{\rho}\right) dP + P d \left(\frac{1}{\rho}\right) \tag{3}$$

But $dQ = dE + dW = dE + pd \left(\frac{1}{\rho}\right)$ and for a unit mass and a single species fluid,

$dQ = Tds$. Therefore we have

$$dE = Tds - pd\left(\frac{1}{\rho}\right) \quad (3.1)$$

In view of (3.1), equation (3.0) yields:

$$dh = Tds + \left(\frac{1}{\rho}\right)dP + pd\left(\frac{1}{\rho}\right) - pd\left(\frac{1}{\rho}\right) \quad (3.2)$$

hence

$$dh = Tds + \left(\frac{1}{\rho}\right)dP \quad (3.3)$$

Assuming that $u \frac{\partial P}{\partial x}$ and $v \frac{\partial P}{\partial y}$ were negligible and $dh = Cp dT$, equation (2.8) reduces to

$$C_p \rho \frac{\partial T}{\partial t} + C_p \rho \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 + q \quad (3.4)$$

Now, the convection equation is expressed as:

$$q = KAdT \quad (3.5)$$

where $dT = (T_\infty - T_s)$ is the difference in temperature between the body surface and the bulk fluid. A is the area of the surface.

In this case, the area of the surface was the length of the curved surface and for this concave surface which had a destabilizing effect, the effect of the curved surface was taken into account by multiplying the area, A by a dimensionless factor earlier defined. This resulted to:

$$q = AfK dT \quad (3.6)$$

Where q is the heat transferred per unit time.

On replacing f, equation (3.6) reduces to

$$q = k \left(1 - \frac{1}{4} \frac{k_r u}{\left(\frac{\partial u}{\partial y}\right)} \right) A(T_\infty - T_s) \quad (3.7)$$

From Newton's law of cooling, the local heat flux is given by

$$q_s'' = h(T_\infty - T_s) \quad (3.8)$$

Where h is the local convection coefficient.

Since the flow conditions varied from one point to another on the curved surface, both q_s'' and h also varied along the curved surface.

For any particular distance x from the edge of the curved surface, q_s'' was found by applying the Fourier's Law to the fluid. This was done at $y = 0$ and was given as:

$$-q_s'' = k \frac{\partial T}{\partial y}, \text{ which can be re-written as:} \quad (3.9)$$

$$q_s'' = -k \frac{\partial T}{\partial y}$$

The local convection heat transfer is expressed as

$$h = -k \frac{\partial T}{\partial y} (T_\infty - T_s)^{-1} \quad (4)$$

At the thermal boundary layer, the rate of heat conduction along the y- direction was larger than that along the x- axis i.e $\frac{\partial T}{\partial y} \gg \frac{\partial T}{\partial x}$

Then we have:

$$C_p \rho \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) + C_p \rho \frac{\partial T}{\partial t} = q + k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 \quad (4.1)$$

From the above approximations, equation (4.1) reduces to

$$C_p \rho \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 + q \quad (4.2)$$

But the value of q is replaced with equation (3.7) in order to take care of the curvature effects and hence on substituting equation (3.7) in equation (4.2) we have:

$$C_p \rho \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 + k \left(1 - \frac{1}{4} \frac{k_r u}{\left(\frac{\partial u}{\partial y}\right)} \right) A(T_\infty - T_s) \quad (4.3)$$

Equation (4.3) gives the equation of energy for convective heat transfer over an immersed curved surface.

3. NON-DIMENSIONALIZING THE EQUATIONS GOVERNING THE FLOW

In our research work, we let L, V, P and T to be the characteristic length, velocity, pressure and temperature respectively. The following transformations are used to reduce our equations in a dimensionless form;

$$\frac{x}{x^*} = L, \quad \frac{y}{y^*} = L, \quad \frac{u}{u^*} = V, \quad \frac{v}{v^*} = V, \quad \frac{p}{p^*} = P,$$

$$T^*(T_\infty - T_s) = T - T_s$$

$$t^*L = tV \text{ Or } t = \frac{t^*V}{L}$$

3.1 Equation of Continuity

For this particular fluid flow, the equation of continuity is given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (4.4)$$

On non-dimensionalizing, the equation of continuity becomes

$$\frac{\partial(u^*V)}{\partial(x^*L)} + \frac{\partial(v^*V)}{\partial(y^*L)} = 0 \quad (4.5)$$

$$\text{Or } \frac{V}{L} \left(\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} \right) = 0 \quad (4.6)$$

$$\text{Or } \left(\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} \right) = 0 \quad (4.7)$$

3.3 The Energy Equation

The equation of conservation of energy is given by

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{c_p \rho} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{c_p \rho} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{kA}{c_p \rho} (T_\infty - T_s) \left(1 - \frac{1}{4} \frac{k_r u}{\left(\frac{\partial u}{\partial y} \right)} \right) \quad (5.2)$$

From the boundary approximations the above equation reduces to

$$\frac{\partial T}{\partial t} = \frac{k}{c_p \rho} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{c_p \rho} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{kA}{c_p \rho} (T_\infty - T_s) \left(1 - \frac{1}{4} \frac{k_r u}{\left(\frac{\partial u}{\partial y} \right)} \right) \quad (5.3)$$

From the non-dimensional form of T, we have:

$$T^* = \frac{T - T_s}{(T_\infty - T_s)}, \text{ which on making T the subject of the formulae yields}$$

$T = T^*(T_\infty - T_s) + T_s$ and thus the equation of energy becomes

$$\frac{\partial [T^*(T_\infty - T_s) + T_s]}{\partial \left(\frac{t^*V}{L} \right)} = \frac{k}{c_p \rho} \frac{\partial^2 [T^*(T_\infty - T_s) + T_s]}{\partial (y^*L)^2} + \frac{\mu}{c_p \rho} \left(\frac{\partial(u^*V)}{\partial(y^*L)} \right)^2 + \frac{kA}{c_p \rho} (T_\infty - T_s) \left(1 - \frac{1}{4} \frac{k_r (u^*V)}{\left(\frac{\partial(u^*V)}{\partial(y^*L)} \right)} \right) \quad (5.4)$$

On further simplification, the above equation yields

$$\frac{V(T_\infty - T_s)}{L} \frac{\partial T^*}{\partial t^*} = \frac{k}{c_p \rho} \frac{(T_\infty - T_s)}{L^2} \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\mu V^2}{c_p \rho L^2} \left(\frac{\partial u^*}{\partial y^*} \right)^2 + \frac{kA}{c_p \rho} (T_\infty - T_s) \left(1 - \frac{1}{4} \frac{k_r u^* L}{\left(\frac{\partial u^*}{\partial y^*} \right)} \right) \quad (5.5)$$

Diving all through by the term $\frac{V(T_\infty - T_s)}{L}$, we obtain

3.2 The Momentum Equation

The equation of conservation of momentum for this flow problem is given by

$$\frac{\partial u}{\partial t} = P_t + v \frac{\partial^2 u}{\partial y^2} + k_r u^2 \quad (4.8)$$

On non-dimensionalizing, the equation becomes:

$$\frac{\partial(u^*V)}{\partial \left(\frac{t^*V}{L} \right)} = P_t^* + v \frac{\partial^2(u^*V)}{\partial (y^*L)^2} + k_r (u^*V)^2 \quad (4.9)$$

On re-arrangement, the equation becomes

$$\frac{V^2 \partial u^*}{L \partial t^*} = P_t^* + \frac{vV}{L^2} \frac{\partial^2 u^*}{\partial y^{*2}} + k_r V^2 u^{*2} \quad (5)$$

Multiplying both sides by $\frac{L}{V^2}$ we have

$$\frac{\partial u^*}{\partial t^*} = \frac{PL}{V^2} P_t^* + \frac{v}{LV} \frac{\partial^2 u^*}{\partial y^{*2}} + k_r L u^{*2} \quad (5.1)$$

This is the equation of momentum in non-dimensional form

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{C_p \rho L V} \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\mu V}{C_p \rho L (T_\infty - T_s)} \left(\frac{\partial u^*}{\partial y^*} \right)^2 + \frac{k L A}{C_p \rho V} \left(1 - \frac{1}{4} \frac{k_r u^{*L}}{\left(\frac{\partial u^*}{\partial y^*} \right)} \right) \quad (5.6)$$

Multiplying the term $\frac{\mu V}{C_p \rho L (T_\infty - T_s)} \left(\frac{\partial u^*}{\partial y^*} \right)^2$ by V in the numerator and the denominator, we obtain

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{C_p \rho L V} \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\mu V \cdot V}{C_p \rho L \cdot V (T_\infty - T_s)} \left(\frac{\partial u^*}{\partial y^*} \right)^2 + \frac{k L A}{C_p \rho V} \left(1 - \frac{1}{4} \frac{k_r u^{*L}}{\left(\frac{\partial u^*}{\partial y^*} \right)} \right) \quad (5.7)$$

The equation (5.7) represent the equation of conservation of energy in non-dimensional form

4. NUMERICAL METHOD OF SOLUTION

In carrying out this study, the governing equations are solved using the finite difference method. We chose the step values $\Delta y = 0.09$ and $\Delta t = 0.2$, in order to bring convergence and consistency in the values to be obtained.

The derivatives in the governing equations are replaced by numerical difference approximations to obtain the equations in finite difference form, written as:

$$U_{i+1,j}^* = \frac{P_{t_i}^* \left(\frac{\Delta t L p}{v^2} \right) + \Delta t (\text{Re})^{-1} \left(\frac{u_{i+1,j+1}^* + u_{i+1,j-1}^* - u_{i,j+1}^* - 2u_{i,j}^* + u_{i,j-1}^*}{2(\Delta y)^2} \right) + u_{i,j}^* + \Delta t L k_r u_{i,j}^{*2}}{\left(1 + \frac{\Delta t}{\text{Re}(\Delta y)^2} \right)}$$

Subject to the boundary conditions:

$$u^*(t^*, 0) = 0$$

$$u^*(t^*, \infty) = 1$$

$$u^*(0, y^*) = y^*$$

and

$$T_{i+1,j}^* = \left(T_{i,j}^* + \Delta t (\text{Pe})^{-1} \left[\frac{T_{i+1,j+1}^* + T_{i+1,j-1}^* + T_{i,j+1}^* - 2T_{i,j}^* + T_{i,j-1}^*}{2(\Delta y)^2} \right] + \frac{\Delta t \text{Ec}}{\text{Re}} \left[\frac{(u_{i+1,j+1}^* - u_{i+1,j}^* + u_{i,j+1}^* - u_{i,j}^*)^2}{[2(\Delta y)]^2} \right] + \frac{AL^2 \Delta t}{\text{Pe}} - \left[\frac{(4pe)^{-1} \Delta t AL^3 K_r u_{i,j}^*}{\left(\frac{u_{i+1,j+1}^* - u_{i+1,j}^* + u_{i,j+1}^* - u_{i,j}^*}{2\Delta y} \right)} \right] \right) \div \left(1 + \frac{\Delta t}{\text{Pe}(\Delta y)^2} \right)$$

Subject to boundary conditions as in below:

$$T^*(t^*, 0) = 0$$

$$T^*(t^*, \infty) = 1$$

$$T^*(0, y^*) = y^*$$

5. RESULTS AND DISCUSSION

5.1 Results

We solved our governing equations and obtained the results which were presented graphically using MATLAB software.

5.2 Discussion

From Fig. 2, when the length of the curvature is increased form L= 0.5 to L= 1.0, the free stream velocity is accompanied by a considerable increase from 0.275501 to 0.360971 as shown on the graph.

This is because as the length of the curvature increases, the velocity gradient also increases. Increase in velocity gradient increases the velocity of the fluid flow in consideration i.e. when the length

of the curvature is increased, the velocity gradient also increases and a consequent increase in free stream velocity is recorded.

More so, when the velocity gradient is increased, the kinetic energy of the fluid particles in motion increases at the boundary layer which implies that the fluid particles are at high velocities

From Fig. 3, we note that when the length of the curvature is increased from L = 0.5 to L = 1.0, the heat dissipation in the boundary layer increases from 0.392678 to 0.572599.

This is because the increase in the length of the curvature increases the velocity gradient which leads to increase in shear stresses. The friction between the fluid particles and the surface in consideration is brought about by these shear stresses. In return, this friction force causes the dissipation of heat in the boundary layer. This is due to the fact that the shear stress is directly proportional to velocity gradient. i.e $\tau = \mu \frac{\partial u}{\partial y}$. when the velocity gradient is increased, the shear stress increases which brings about friction between the fluid particles leading to increase in heat dissipation.

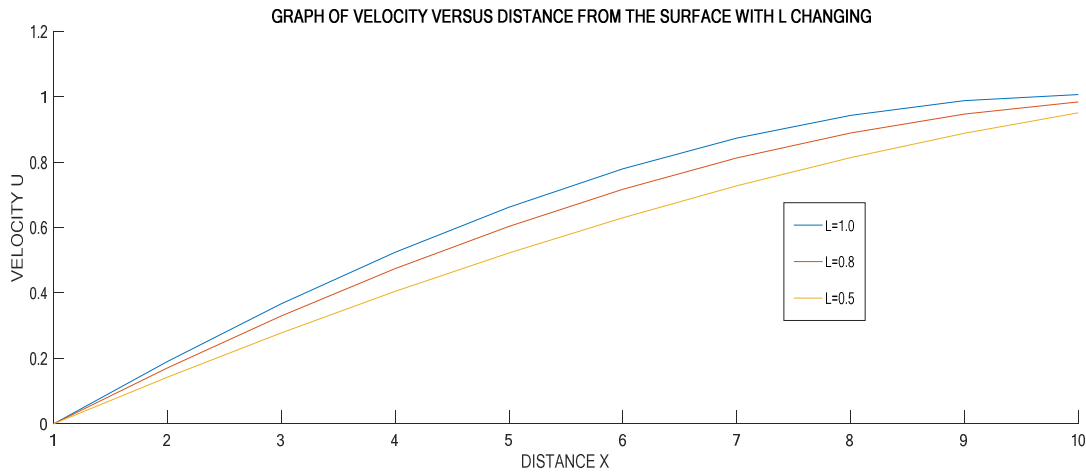


Fig. 2. Velocity profiles for Re = 1.3, Pe = 1, V=1, Kr = 1, Ec = 2, A = 2, Pt = 1

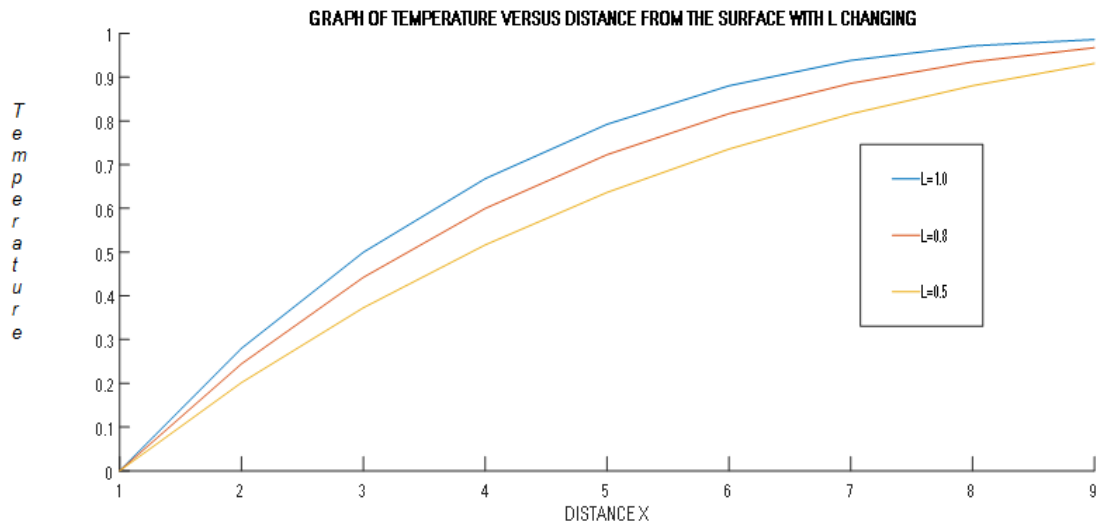


Fig. 3. Velocity profiles for Re=1.3, Pe=1, V=1, Kr = 1, Ec = 2, A = 2, Pt = 1

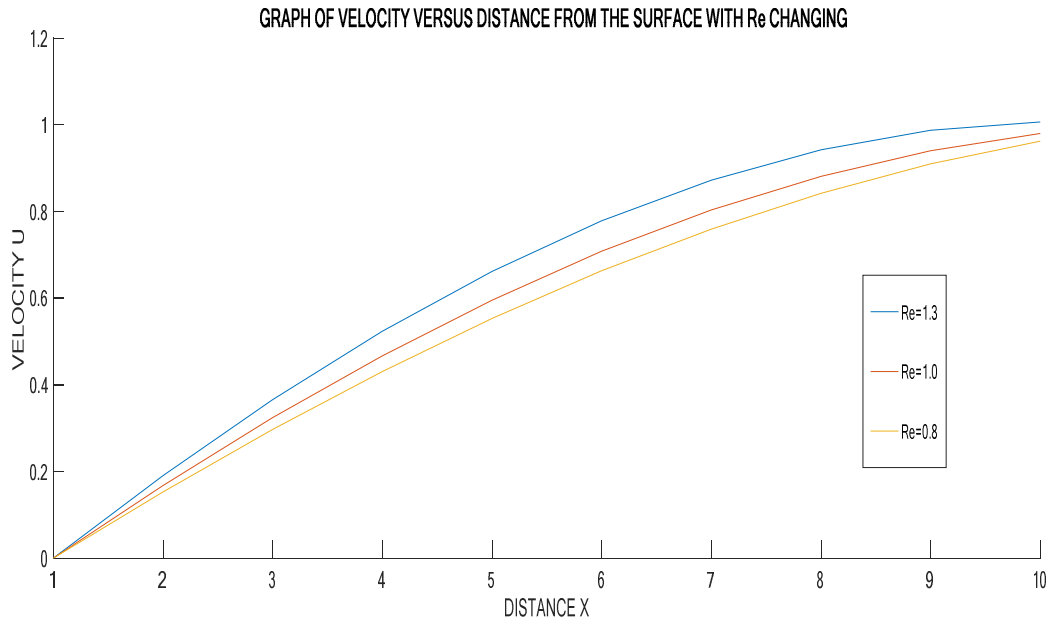


Fig. 4. Velocity profile for $L=1$, $Pe=1$, $V=1$, $Kr=1$, $Ec=2$, $A=2$, $Pt=1$

From Fig. 4, we note that as the Reynolds number increases from 0.7 to 1.3, a direct consequence of the increase in inertia forces occurred leading to an increase in velocity from 0.297405 to 0.367155. When the Reynolds number is large, the inertia forces tend to be dominated over the viscous force and

consequently, the friction of the fluid particles and the surface in consideration is very minimal resulting to an increase in velocity of the fluid flow. At large inertia forces, the velocity of the fluid is high since low viscous forces implies that little or minimal friction exists between the fluid particles and the surface in consideration.

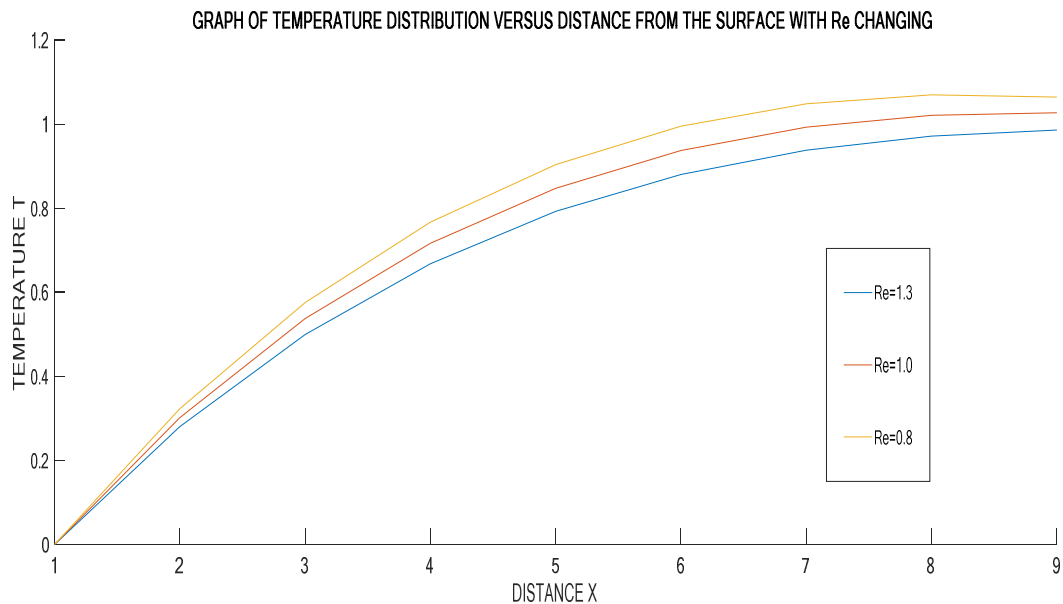


Fig. 5. Temperature profile for $L=1$, $Pe=1$, $V=1$, $Kr=1$, $Ec=2$, $A=2$, $Pt=1$

From Fig. 5, we note that when the Reynolds number is increased from 0.8 to 1.3, the heat dissipation in the boundary layer reduces from 0.613144 to 0.508381.

This is because when the value of the Reynolds number is low, the inertia forces are minimal. The viscosity of the fluid thus dominates over the inertia forces and consequently, the friction of the fluid particles with surface increases resulting to increase in heat dissipation within the boundary layer. When Reynolds number is large, the viscous forces are very minimal since inertia forces dominate in the fluid flow. Consequently, the friction of the fluid particles with the surface is minimal and this results to the minimal dissipation of heat within the boundary.

6. CONCLUSION AND RECOMMENDATIONS

When the length of the curvature was increased, this led to velocity and temperature rise. This matched the theoretical explanation since the increase in velocity gradient increases the velocity of the fluid flow. Also at high velocity gradients, the shear stresses are high which brings about the friction between the fluid particles and the surface. Consequently, heat is dissipated. It thus follows that the length of the curvature is directly proportional to the velocity and temperature distribution.

It is also observed that at large Reynolds number, the inertia forces are large compared to the viscous effect of the fluid and consequently, the fluid velocity increases. This is in line with a theoretical explanation since at low viscosity, minimal shear stresses exist between the fluid particles and the surface and thus the velocity of the fluid is favoured. At low Reynolds number, the viscosity of the fluid is high since there are minimal inertia forces. Consequently, the fluid velocity goes down. At large Reynolds number, the amount of heat dissipated at the boundary layer is minimal due to minimal friction between the fluid particles and the surface.

It, therefore, follows that Reynolds number is directly proportional to the velocity distribution and inversely proportional to the temperature distribution in the boundary layer.

It is recommended that further investigations be done in the following areas:

1. Compressible fluid over immersed surface
2. Convective heat transfers on turbulent fluid flows over the immersed curved surface
3. Study of the same orientation but in three-dimension

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Gathungu DK. Analysis of convective heat transfer in a fluid flow over an immersed axi-symmetrical body with curved surfaces. *Journal of Engineering and Applied Science*; 2011.
2. Gupta BK. Effects of longitudinal surface curvature on heat transfer with dissipation. *Journal of Heat and Mass Transfer*. 2003; 14:1575-1588.
3. Bradshaw P. The analogy between streamline curvature and buoyancy in turbulent shear flow. *Journal of Fluid Mechanics*. 2006;36:177-191.
4. Gordon CE. Numerical solutions of partial differential equations Gaithersburg. Maryland; 2010.
5. Mugambi KE. An investigation forces produced by fluid motion on a submerged finite plate. An Unpublished Msc. Thesis; Jomo Kenyatta University of Agriculture and Technology, Nairobi, Kenya. 2008;1-9.
6. George OO. A study on convective heat transfer in a fluid flow over an immersed curved surface. An Unpublished Msc. Thesis; Jomo Kenyatta University of Science and Technology, Nairobi, Kenya. 2009;1-33.
7. Barenblatt GT, Chorin AJ, Protokishin, VM. A model of a turbulent boundary layer with a non-zero pressure gradient. *Proceedings of National Academy of Science U.S.A.* 2002;5772-5776.
8. Fukagata K, Iwamoto K, Kasagi N. Contribution of the reynolds stress distribution to the skin friction in wall bounded flows. *Journal of Physics of Fluids*. 2002;14:73-76.

Appendix 1

Computer code

```

% solution of both velocity and momentum equation is described here. where
% u1 is the solution for velocity equation and u2 is the solution to
% momentum equation. the varying parameters are also described here.
p(1) = 1.0; % Reynolds number
p(2) = 1; % peclet number
p(3) = 1; % pressure
p(4) = 1; % length
p(5) = 1; % velocity
p(6) = 1; %kr radius of curvature
p(7) = 2; % eckert number
p(8) = 2; % surface area of the curvature
p(9) = 1; % pt
m = 0;
x = linspace(0,1,11);
t = linspace(0,1,5);
sol = pdepe(m,@pdex2pde,@pdex2ic,@pdex2bc,x,t,[],p);
u1 = sol(:,1);
u2 = sol(:,2);
figure(1)
hold on
%surf(x,t,u1);
hold on
plot(u1(4,1:10))
figure(2)
hold on
%surf(x,t,u2);
plot(u2(2,1:9))

function [ c,f,s ] = pdex2pde( x,t,u,DuDx,p )
% PARTIAL DIFFERENTIAL EQUATION
% Both velocity and momentum equation are described here. u(1)
% representing the velocity equation values while u(2) will be representing momentum equation
values
global Re Pe P L V Kr Ec A Pt
Re = p(1); Pe = p(2); P = p(3); L = p(4); V = p(5); Kr = p(6); Ec = p(7); A = p(8); Pt = p(9);
c = [1; 1];
f = [1/Re; 1/Pe].*DuDx;
s = [(P*L/V^2)*Pt+Kr*L*u(1); (Ec/Re)*(DuDx(1))^2+(L^2*A/Pe)*(1-1/4*((Kr*u(1))*L*DuDx(1)^(-1)))]);
end

function [ u0 ] = pdex2ic( x,p )
% INITIAL CONDITION
% U0 represents the initial condition for both velocity and momentum
% equation. The intial condition is dependent on distance x
u0 = [x; x];
end

```

```
function [ pl,ql,pr,qr ] = pdex2bc( xl,ul,xr,ur,t,p )
%BOUNDARY CONDITION
% Boundary conditions for both velocity and momentum equations are
% described here
pl = [ul(1); ul(2)];
ql = [0; 0];
pr = [ur(1)-1; ur(2)-1;];
qr = [0; 0];
end
```

© 2018 Nyaga et al.; This is an Open Access article distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/4.0>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Peer-review history:
The peer review history for this paper can be accessed here:
<http://www.sciencedomain.org/review-history/26156>