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Can the Distance–Redshift Relation be Determined from Correlations between Luminosities?

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Abstract

We explore whether an independent determination of the distance–redshift relation, and hence cosmological model parameters, can be obtained from the apparent correlations between two different wave-band luminosities or fluxes, as has been claimed in recent works using the X-ray and ultraviolet luminosities and fluxes of quasars. We show that such an independent determination is possible only if the correlation between luminosities is obtained independently of the cosmological model and measured fluxes and redshifts, for example, based on sound theoretical models or unrelated observations. In particular, we show that if the correlation is determined empirically for two luminosities obtained from fluxes and redshifts, then the method suffers from circularity. In the case where the observed correlation between fluxes in very narrow redshift bins is used as a proxy for the luminosity correlation, we show that one is dealing with a pure tautology with no information on distances and cosmological model. We argue that the problem arises because of the incomplete treatment of the correlation, and we use numerical methods with a joint X-ray and ultraviolet quasar data set to demonstrate this shortcoming.

Unified Astronomy Thesaurus concepts: Cosmological models (337); X-ray quasars (1821)

1. Introduction

Recently, Risaliti & Lusso (2019)—hereafter RL—used a measure of the nonlinear correlation between the X-ray and ultraviolet (UV) luminosities of quasars to arrive at a determination of the shape of the luminosity–distance function in the redshift range $1.4 < z < 5$, finding that it deviates from that of the ΛCDM cosmology. This deviation favored a larger overall matter density fraction Ω_m and an evolving dark energy equation of state. Similar results were obtained by the same method in subsequent works using quasar samples at higher redshifts (Salvestrini et al. 2020), incorporating additional X-ray quasar catalogs (Sacchi et al. 2022; Lusso et al. 2020; Bisgoni et al. 2021) and joint analyses of quasars along with other cosmological probes (Bargiacchi et al. 2022). These findings, if true, would be an important new procedure for using extragalactic sources with wide dispersion in their luminosity (i.e., sources far from being a "standard candle") for precision cosmological studies on par with near-standard candles like Cepheid variables and Type Ia supernovae, but extending to higher redshifts.

The main aim of this Letter is a close scrutiny of the basics of the procedure proposed by RL and to point out some of its shortcomings. Before getting into details, it is important to emphasize several crucial aspects of the procedure.

The first is that this is a purely phenomenological method of using the correlation between observed fluxes, or deduced apparent luminosities, that have no direct or obvious relation to the astrophysics of the sources. That this is true for a correlation between fluxes is obvious. As has been pointed out in many publications, it is also true for an apparent correlation between luminosities (*L*–*L* correlation). Using nonparametric methods developed by Efron & Petrosian (1992) (EP) in several publications (e.g., Singal 2013; Singal et al. 2016; Zeng et al. 2021), we have shown that the observed *L*–*L* correlations (and luminosity and redshift distributions) quantitatively and qualitatively are very different from intrinsic ones due to (1) multidimensional observational selection effects that truncate the data, (2) the common dependence on the calculated luminosities on distance or redshift, and (3) possible difference in the redshift evolution of the luminosities in different wave bands. In a recent work (Singal et al. 2019), using analytic methods and simulations, we explored in depth to what extent apparent correlations in multiwavelength flux-limited data are indicative (or not) of intrinsic correlations and hence the physics of the accretion disks, jets, and characteristics of supermassive black holes, confirming the above findings with actual data. In a more recent paper (Singal et al. 2022), we find similar differences between the observed and intrinsic *L*–*L* correlation in the X-ray and UV wave bands (the particular wave bands used by RL). An analysis, also using the EP method, by Dainotti et al. (2022) agrees qualitatively with this result with a somewhat different luminosity evolution rate in the X-ray band. However, it should be emphasized that the phenomenological method used by RL is independent of such differences between the apparent and intrinsic *L*–*L* correlation. As will be clear from the description of the method given below, one could use either correlation leading to the same result.

The second is that even in the phenomenological approach one has to include the effects of the flux truncation (induced by observational selections process) on the observed correlations.

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This could change the result only quantitatively but not qualitatively. Consequently, we will ignore this effect.

Finally, as mentioned above, X-ray/UV (L–L) correlation is not unique. Quasars and other active galactic nuclei (AGNs) show similar nonlinear L–L correlations in other pairs of wave bands: radio–optical (e.g., Singal 2013), mid-infrared–optical (e.g., Singal et al. 2016), and optical–gamma-ray (Zeng et al. 2021). Thus, if there is any deviation from ΛCDM in one pair of wave bands, the same should be true in other pairs.

In this work. we explore the question of whether the correlation between two wave-band luminosities (or observed fluxes) can be used at all to reliably achieve an independent determination of the distance–redshift relation as was done in RL and subsequent works. In Section 2, we discuss the potential logical issues with such a method. In Section 3 we use some numerical analysis to quantify the reasons for the apparent deviation obtained by RL and explore how the procedure can lead to misleading results. A brief summary and some discussion is presented in Section 4.

2. Analytic Considerations of the Basic Problem

The RL method relies on the assumption that if the degree and form of the correlation between the luminosities in two different wave bands (in their case, the X-ray and UV bands) can be deduced, then given observed fluxes in the two bands and redshifts, one can determine the dependence of the luminosity distance function $D_L(z)$ on redshift. To explore the validity of this method, we use two different possible approaches to the problem.

A. Starting from L–L correlation:

Following RL, let us assume that the L–L correlation can be expressed with a power law with index γ_L and (the dimensionless) proportionality constant B_L as[4]

$$L_{\rm x}(z) = B_L \cdot (L_{\rm UV}(z))^{\gamma_L}. \quad (1)$$

An important requirement of the procedure proposed by RL is that γ_L and B_L are independent of the cosmological model; they could depend on redshift but not cosmological parameters Ω_M, Ω_Λ, etc. To start with, following RL, we assume that these parameters are constants (i.e., independent also of redshift), but as will be evident from the formalism presented below, this assumption is not actually required for the proposed procedure to work. The luminosity–flux relation for a generic wave band a is

$$F_a = \frac{L_a K_a(z)}{4\pi D_L^2(z)}, \quad (2)$$

where $D_L(z)$ is the luminosity distance and $K_a(z)$ is the K-correction factor.[5] To simplify the algebra for the moment, we can ignore the small effect of the K-correction or use rest-frame fluxes, $F_r(z) = F(z)/K(z)$, at a well-defined frequency band (or monochromatic flux). Then, substituting this in Equation (1), it is easy to show that we obtain the redshift dependence of the luminosity distance as

$$4\pi D_L(z)^2 = \left(\frac{B_L\, F_{\rm UV}^{\gamma_L}(z)}{F_{\rm x}(z)}\right)^{(1-\gamma_L)^{-1}}, \quad (3)$$

which, as stressed above, would be the case even when the fit parameters B_L and γ_L depend on redshift but not the cosmological model. RL then used the measured fluxes in the X-ray and UV bands to determine the luminosity distance as a function of redshift.[6]

However, there are many unsupported assumptions in this procedure, the most important of which is that the correlation form in Equation (1) is independent of a cosmological model. In general, the L–L correlations are usually based on luminosities determined from observed (or rest-frame) fluxes using Equation (2), which requires an assumed cosmological model that gives the required luminosity distance function $D_L(z)$ (as was done to plot the luminosity versus redshift in Figure 1 of RL). In that case, it is obvious that the above procedure is logically circular and the method should return the assumed form of the luminosity distance used in calculating the luminosities, modulo observational uncertainties, numerical errors, and neglect of truncation effects.

If one uses the luminosity evolution (LE)-corrected luminosities $L'_a = L_a/g_a(z)$, where $g_a(z)$ describes the LE in wave band a, one could follow the above steps using the intrinsic (or de-evolved) L'–L' correlation, with $B'(z)$ and $\gamma'(z)$ possibly not equal to $B(z)$ and $\gamma(z)$. This leads to the same final results (Equation (3)) with the addition of the LE factors of $g_{\rm x}(z)$ and $g_{\rm UV}(z)$, but still suffering from the same circularity because one must assume a form for $D_L(z)$ to determine the LE functions and the intrinsic correlation γ'_L. Dainotti et al. (2022) use this (circular) procedure and obtain a similar result to RL. This is probably because the LE factors they obtain for the X-ray and UV bands are similar, which yields $\gamma'(z) \sim \gamma(z)$. See also the discussion below.

B. Starting from the F–F correlation:

On page 2 of their paper, RL indicate that their calculation of the correlation index γ is "cosmologically independent"—i.e., independent of a specific cosmological model and therefore of the luminosity distance—which must mean that in their figure showing γ versus redshift (supplementary Figure 16), the correlation indices are obtained using the correlation between fluxes, $F_{\rm UV}$ and $F_{\rm x}$, in redshift bins z_i to $z_i + \Delta z_i$ assuming a power-law correlation:

$$F_{\rm x} = B_f(z) \cdot (F_{\rm UV})^{\gamma_f(z)}. \quad (4)$$

In general, both the proportionality constant B_f and index γ_f could vary with redshift. In Figure 8 of the supplementary material, RL show variation of both quantities with redshift. There are small variations of γ_f, with an average value of $\gamma_f = 0.607 \pm 0.05$, and $\beta_f(z) = \log B_f(z)$ increasing with redshift almost monotonically from 3.8 to 4.5 over the redshift range $z = 0$–4.[7] RL use a constant value for γ_f, but it is not clear

[4] We express the luminosities in units of some fiducial luminosity, L_0, whose value is irrelevant but renders the proportionality constant B_L dimensionless. Note also that RL describe the L–L correlation in log space, obtaining γ as the slope of the linear regression fit and intercept $\beta_L = \log B_L$.

[5] In some publications (e.g., Bloom et al. 2001), the inverse of this is defined as the K-correction. We use the original definition given by Oke & Sandage (1968) for galaxies.

[6] It is clear that the method fails for a linear correlation, $\gamma_L = 1$, thus the requirement of nonlinearity. In what follows, we assume that $\gamma_L < 1$, which seems to be the case of interest here, using L_X as the dependent variable. In the opposite case, one would have $\gamma_L > 1$.

[7] There is some apparent ambiguity as to whether RL use a value of $\gamma = 0.633$ as seems to be indicated in their main paper or $\gamma = 0.607$ as indicated in their supplementary paper, but the precise value is not important for our analysis.

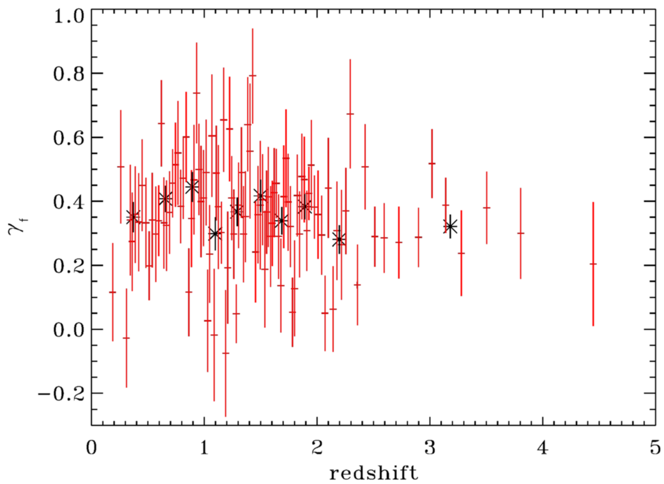

Figure 1. A determination of the best-fit power-law correlation between the X-ray and UV fluxes of quasars, in the form of Equation (4), as determined for the data set assembled in Singal et al. (2022). Results, with statistical uncertainties, are shown for both 10 (stars) and 100 (red pluses) bins of redshift with equal numbers of objects, in respective bins. The average value in each case is close to $\gamma_f = 0.36$. These results evidence a significantly larger dispersion than those obtained by RL.

if the same is assumed for $B_f(z)$, or whether its variation with redshift is included in the analysis. As will be clear, the exact values of these parameters and whether they are constant or vary with redshift, do not change the arguments presented below. In fact, using a larger sample of quasars analyzed in Singal et al. (2022), we find large variation of $\gamma_f(z)$ (especially for small redshift bins) with a smaller mean of 0.36 shown in Figure 1.

Starting from Equation (4), it is important to note that correlation parameters calculated from fluxes cannot be substituted for correlation parameters needed for luminosities because the L–F relation depends on $D_L^2(z)$.

Let us first consider the relation between the two power-law indexes, $\gamma_f(z)$ and $\gamma_L(z)$ (which for maximal generality we will let vary with redshift) and the possibility that $\gamma_L(z) = \gamma_f(z)$ independently of the cosmological model. The first possibility is if γ_f's are obtained using infinitesimally small redshift bins, within which the variation of $D_L^2(z)$ is small and can be ignored. From Figure 16 in their supplementary paper, it appears that RL are using 10 bins spanning the redshifts from 0 to 4, with the largest bin spanning from $z = \sim 2.7$ to $z = 4$. The above assumption is clearly not valid for this bin with $\Delta D_L^2 / \langle D_L^2 \rangle \sim 0.8$ (and not negligible), where the claimed deviation of the luminosity distance model from ΛCDM is most significant.

The second possibility is that we have prior knowledge that the LE in the two bands are identical. In that case, the L–L correlation will be a scaled (by $4\pi D_L^2(z)$) version of the F–F correlation, yielding $\gamma_L(z) = \gamma_f(z)$. We note that RL and many subsequent papers ignore the possibility of LE. As mentioned above, Dainotti et al. (2022) find somewhat similar (but not identical) LEs for the X-ray and optical–UV bands, indicating that this equality is approximately true. However, in Singal et al. (2022), using a more rigorous accounting of the X-ray flux threshold, we obtain very different LEs.

But even assuming that these uncertainties can be ignored, let us follow RL and set $\gamma_L(z) = \gamma_f(z) = \gamma(z)$, assuming that $\gamma_L(z)$ is independent of the cosmological model, and consider the relation between the two proportionality constants. If we replace the fluxes in Equation (4) by luminosities using Equation (2), still ignoring the K-corrections, we obtain

$$L_x(z) = B_f(z) \times L_{\rm UV}^{\gamma(z)}(z) \times [4\pi D_L^2(z)]^{1-\gamma(z)}. \quad (5)$$

Comparing this with the required relation in Equation (1), we find

$$B_L(z) = B_f(z) \times [4\pi D_L^2(z)]^{1-\gamma(z)}, \quad (6)$$

which clearly is not independent of the luminosity distance and the cosmological model, a strict requirement of the procedure to yield an independent determination of the luminosity distance function. If now one substitutes this form of $B_L(z)$ in Equation (3), the luminosity distance $D_L^2(z)$ simply cancels out and one recovers Equation (4), which is where we started from. Thus, we conclude that we are dealing with pure tautology and that there is no logical procedure for recovering $D_L(z)$. We emphasize again that this conclusion is true whether or not $\gamma(z)$ varies with redshift or is actually a constant, and even if one has infinitesimally small redshift bins and identical LE in the two bands so that the assumption $\gamma_L(z) = \gamma_f(z) = \gamma(z)$ is valid.

In summary, the procedure proposed by RL is either logically circular or pure tautology, leading to the inevitable conclusion that it is not possible to arrive at an independent determination of the luminosity distance function $D_L(z)$ using either the correlation index γ_L calculated from the luminosities or γ_f calculated from the fluxes. The former has an inescapable a priori dependence on the form of $D_L(z)$, resulting in a logical circularity, and the latter has no relation to $D_L(z)$.

One may ask why, in light of the circular or tautological aspects discussed above, did RL not obtain the $D_L(z)$ function of the ΛCDM cosmology (or fail numerically) when they carried out their analysis. We explore this issue in the next section.

3. Exploring the RL Method with Quasar Data

Given the considerations of Section 2, any analysis that attempts to use flux–redshift data to determine $D_L(z)$ based on Equation (3) should return the luminosity distance function of the assumed cosmological model (here ΛCDM). However, RL and subsequent works (e.g., Khadka & Ratra 2020; Dainotti et al. 2022) obtained a $D_L(z)$ function that deviates from that of the ΛCDM cosmology, favoring a larger value for the matter density Ω_m and an evolving dark energy equation of state.

The formalism presented above indicates that there is a degeneracy between the assumed redshift dependencies of D_L and the correlation parameters B and γ, irrespective of whether we start with Equations (1) or (4). It is clear from Equations (3) and (5) (or (6)) that this degeneracy takes the form of

$$D_L(z) \propto B_f(z)^{\frac{1}{2(\gamma(z)-1)}}. \quad (7)$$

This indicates that an important source of this discrepancy found by RL is that they did not include the redshift dependence of $B_f(z)$.

As the above equation shows, an increase with z of $B_f(z)$, and/or decrease of $\gamma(z)$ (for $\gamma(z) < 1$), can give rise to a value for $D_L(z)$ that is increasingly lower at higher z's than the true luminosity distance. In the opposite case, one would get a higher $D_L(z)$. For example, an increase of $\beta(z) = \log B_f(z)$ from 3.8 to 4.5 (∼20%) obtained by RL, mentioned above, will

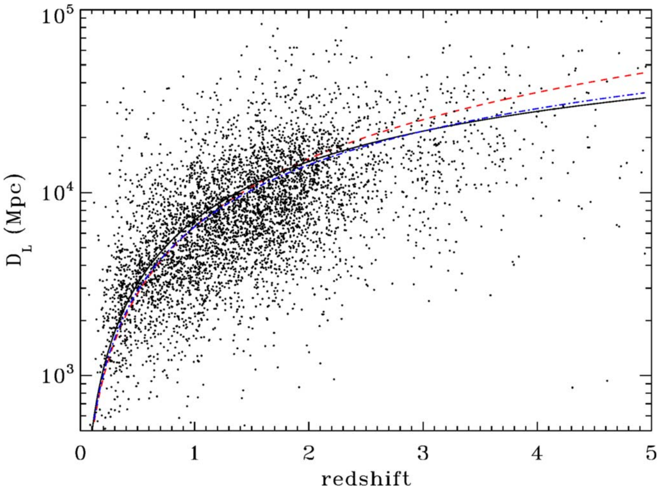

Figure 2. $D_L(z)$ as would be determined by Equation (3), the method of RL, utilizing an X-ray and UV data set of quasars analyzed in Singal et al. (2022). The points show $D_L(z)$ for individual quasars as determined for $\gamma = 0.28$ obtained by Singal et al. (2022), while the solid black line shows the best-fit curve of the form Equation (8) for these points. The dashed–dotted blue line is the result reported by RL for $\gamma = 0.633$. The dashed red line shows $D_L(z)$ for ΛCDM. As evident, a lower value of γ yields a larger deviation from ΛCDM at high z, as discussed in Section 3.

cause a ∼25% decrease in D_L for $\gamma \sim 0.6$, which is about what RL obtained.

To further demonstrate this effect, in Figure 2 we compare the luminosity distance as would be obtained by Equation (3) for two values of the index γ: $\gamma = 0.633$ obtained by RL and $\gamma = 0.28$, which we obtain using the intrinsic $L' - L'$ correlation obtained in Singal et al. (2022). For this purpose, following RL, we assume no redshift dependence for both B_f and γ. As in RL, we fit the determined $D_L(z)$ for $\gamma = 0.28$ points to a third-order polynomial in $\ln(1 + z)$, fixing the terms so that it approaches $D_L(z) = (c/H_0)z$ for $z \ll 1$:

$$D_L(z) = \ln(10)\,(c/H_0)(x + a_2x^2 + a_3x^3),\quad \text{with } x = \log(1 + z). \tag{8}$$

Figure 2 shows the result of this fit as the black curve. We also show the $D_L(z)$ curve obtained by RL for $\gamma = 0.63$ (dashed–dotted blue line) and for ΛCDM (red dashed). A larger deviation from ΛCDM is obtained (at high z) for a smaller value of γ (for $B(z)$ increasing with z) as predicted by Equation (7). An additional possible source of the apparent deviation in $D_L(z)$ from that of ΛCDM, investigated by Yang et al. (2020) and Banerjee (2021), is that the polynomial expansion of Equation (8) generically fails to recover flat ΛCDM beyond $z \sim 2$. Furthermore, Khadka & Ratra (2020) have shown that the deviation in $D_L(z)$ from that of ΛCDM in RL is not as statistically significant as claimed.

4. Summary and Discussion

We find in Section 2 that the method of determining the luminosity distance as a function of redshift (and hence cosmological parameters) from the observed (nonlinear) correlation between two luminosities or fluxes in a population quasars, proposed by RL and utilized in the subsequent works, Salvestrini et al. (2020), Sacchi et al. (2022), and Bisgoni et al. (2021), suffers from a fatal logical inconsistency inherent in the method. It is either circular or tautological.

Determinations of the luminosity distance function commonly involve establishing a correlation between (i) a distance-independent characteristic, such as the oscillation periods of Cepheid variable stars or the decay times of Type Ia supernovae, and (ii) a distance-dependent quantity, such as the luminosity in a wave band (which is distance dependent via the flux–luminosity relation). If such correlations are redshift, or in general, distance, independent, then one can use the "Hubble diagram" to get the luminosity distance, otherwise one needs to take into account how the relevant parameters may change with redshift. This condition is more difficult to establish in attempts to use gamma-ray bursts utilizing relationships between the peak spectral energy and the total energy or peak luminosity (e.g., Yonetoku et al. 2004; Amati et al. 2008). The method of Risaliti & Lusso (2019) and subsequent works is significantly different as it attempts to use the nonlinear correlation between two distance-dependent quantities, namely luminosities in two different wave bands (here X-ray and UV). If valid, this would be a new revolutionary method of determining cosmological parameters from extragalactic sources, such as quasars, or more general AGNs, with rich observations at many different wave bands from radio to gamma-rays.

We have argued that the only way the proposed method would work is if the exact form and parameters of correlation between two luminosities is determined independently of the cosmological model (and hence $D_L(z)$) and without use of the observed fluxes, such as would be obtained from a purely theoretical physical model of the emitting system. If the L–L correlation is obtained, as is commonly the case, from measured fluxes and redshifts, which require knowledge of $D_L(z)$, then the method suffers from circularity and should return the assumed $D_L(z)$, modulo observational and numerical errors, and other uncertainties. An alternative approach of simultaneous optimization over both cosmological parameters and luminosity correlation parameters as was done by Khadka & Ratra (2021a) and Khadka & Ratra (2021b) is a promising possibility, which apparently leads to no statistically significant deviation from ΛCDM.

However, RL obtain correlation parameters using redshift-binned flux–flux correlations. They assume a power-law correlation with two parameters: the power-law index $\gamma_f(z)$ and the proportionality parameter $B_f(z)$ in Equation (4), both of which show some variation with redshift in their analysis. The basic idea they propose is that given infinitesimally small redshift bins, within which the $D_L(z)$ change is negligible, then one can use these parameters for the same form of the L–L correlation with $\gamma_L(z) = \gamma_f(z)$ and $B_L(z) = B_f(z)$. As we discuss in Section 2, there are several problems with this procedure:

1. Some redshift bins used by RL span a wide range of redshift and cannot be consider infinitesimal. As shown in Figure 1, using smaller bins one obtains much larger variation of γ_f than the 10 bins used by RL. Additionally we obtain a significantly different value for γ_f than RL in our analysis with a joint X-ray and UV data set of quasars.
2. RL ignore the small variation of γ_f shown by their data. Nevertheless, let us follow RL and ignore this variation and set $\gamma_L = \gamma_f = \gamma$.
3. They also seem to ignore the relatively significant monotonic variation they obtain for $B_f(z)$, and, most importantly, assume that the same is true for $B_L(z)$. As we

have shown, this is manifestly not well founded and when changing variables from fluxes to luminosities (related through $D_L(z)$), one gets a proportionality constant $B_L(z)$ that depends on $D_L(z)$ (independent of whether the variation of $B_f(z)$ is significant or not) in a way that $D_L(z)$ drops out of the relation, making the argument pure tautology.

Finally, in Section 3 we show that the likely reason RL obtained a luminosity distance function that deviates from the ΛCDM model is related to the fact that they ignore the redshift dependence of $B_f(z)$ and additionally, because of a potentially questionable method of fitting of D_L to redshift. As we demonstrate, the deduced $D_L(z)$ depends on $B_f(z)$ and $\gamma(z)$ (see Equation (7)) in a way that explains the deviation downward from the ΛCDM model that they find at high z, shown in Figure 2. It is our conclusion then that the procedure proposed by RL is not an appropriate method for constraining the luminosity distance function or cosmological parameters.

This work relies in part on data analyzed in Singal et al. (2022), which was obtained from the Sloan Digital Sky Survey (SDSS), and the Chandra and XMM-Newton X-ray observatories. Funding for the SDSS and SDSS-II has been provided by the Alfred P. Sloan Foundation, the Participating Institutions, the National Science Foundation, the U.S. Department of Energy, the National Aeronautics and Space Administration, the Japanese Monbukagakusho, the Max Planck Society, and the Higher Education Funding Council for England. The SDSS Web Site is http://www.sdss.org/. This research has made use of data obtained from the Chandra Source Catalog, provided by the Chandra X-ray Center (CXC) as part of the Chandra Data Archive. This research has made use of data obtained from the 4XMM XMM-Newton Serendipitous Source Catalog compiled by the 10 institutes of the XMM-Newton Survey Science Centre selected by ESA.

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References

Amati, L., Guidorzi, C., Frontera, F., et al. 2008, MNRAS, 391, 577
Bargiacchi, G., Benetti, M., Capozziello, S., et al. 2022, MNRAS, 515, 1795
Banerjee, A., Ó Colgáin, E., Sasaki, M., Sheikh-Jabbari, M. M., & Yang, T. 2021, PhRvB, 818, 136366
Bisgoni, S., Lusso, E., Civano, F., et al. 2021, A&A, 665, A109
Bloom, J., Frail, D., & Sari, R. 2001, AJ, 121, 2879
Dainotti, M. G., Bardiacchi, G., Lenart, A. Ł., et al. 2022, ApJ, 931, 106
Efron, B., & Petrosian, V. 1992, ApJ, 399, 345
Khadka, N., & Ratra, B. 2020, MNRAS, 497, 263
Khadka, N., & Ratra, B. 2021a, MNRAS, 502, 6140
Khadka, N., & Ratra, B. 2021b, MNRAS, 510, 2753
Lusso, E., Risaliti, G., Nardini, E., et al. 2020, A&A, 642, A150
Oke, J. B., & Sandage, A. 1968, ApJ, 154, 210
Risaliti, G., & Lusso, E. 2019, NatAs, 3, 272
Sacchi, A., Risaliti, G., Signorini, M., et al. 2022, A&A, 663, L7
Salvestrini, F., Risaliti, G., Bisogni, S., Lusso, E., & Vignali, C. 2020, A&A, 631, 120
Singal, J., George, J., & Gerber, A. 2016, ApJ, 831, 60
Singal, J., Mutchnick, S., & Petrosian, V. 2022, ApJ, 932, 111
Singal, J., Petrosian, V., Malik, S., & Haider, J. 2019, ApJ, 877, 1
Singal, J., Stawarz, Ł., Lawrence, A., & Petrosian, V. 2013, ApJ, 764, 43
Yang, T., Banerjee, A., & Ó Colgáin, E. 2020, PhRvD, 102, 123532
Yonetoku, D., Murakami, T., Nakamura, T., et al. 2004, ApJ, 609, 935
Zeng, H., Petrosian, V., & Yi, T. 2021, ApJ, 913, 120